

Equilibrium Tuition, Applications, Admissions and Enrollment in the College Market*

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Abstract

I develop and estimate a structural equilibrium model of the college market. Students, having heterogeneous abilities and preferences, make college application decisions, subject to uncertainty and application costs. Colleges, observing only noisy measures of student ability, choose tuition and admissions policies to compete for more able students. Tuition, applications, admissions and enrollment are joint outcomes from a subgame perfect Nash equilibrium. I estimate the structural parameters of the model using data from the NLSY97, via a three-step procedure to deal with potential multiple equilibria. Counterfactual experiments show that first, a perfect measure of student ability would lead to higher enrollee ability and higher average student welfare. Second, funding cuts to public colleges would lead to tuition increases in all colleges and large student welfare loss. Finally, college enrollment would increase by only 2.1% if the lower-ranked public colleges were expanded to accommodate all who applied.

Keywords: College market, tuition, applications, admissions, enrollment, discrete choice, market equilibrium, multiple equilibria, estimation

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1 Introduction

Both the level of college enrollment and the composition of college student bodies continue to be issues of widespread scholarly interest as well as the source of much public policy debate. In this paper, I develop and structurally estimate an equilibrium model of the college market. It provides insights into the determination of the population of college enrollees and permits quantitative evaluation of the effects of counterfactual changes in the features of the college market. The model interprets the allocation of students in the college market as an equilibrium outcome of a decentralized matching problem involving the entire population of colleges and potential applicants.¹ As a result, counterfactuals that directly involve only a subset of the college or student population can produce equilibrium effects for all market participants. My paper thus provides a mechanism for assessing the market equilibrium consequences of changes in government policies on higher education.

While the idea of modeling college matching as a market equilibrium problem is not new, this paper makes advances relative to the current literature by simultaneously modeling three aspects of the college market that are plausibly regarded as empirically important and incorporating them into the empirical analysis.^{2,3} The three aspects are: 1) Application is costly to the student. In addition to application fees, a student has to spend time and effort gathering information and preparing application materials and may incur psychic costs during the process. 2) Students differ in their abilities and preferences for colleges.⁴ 3) Colleges can only observe noisy measures of student ability, and both sides of the market face uncertainties. For the student, admissions are uncertain, which, together with the cost of application, leads to a non-trivial portfolio problem for her: how many and which, if any, colleges to apply to? For the college, the yield of each admission and the quality of a potential enrollee are both uncertain. The inference of these has to account for students' strategies. Colleges' policies are also interdependent because students' application portfolios and their enrollment depend on the policies of all colleges.

I model three stages of the market. First, colleges simultaneously announce their tuition. Second, students make application decisions and colleges simultaneously choose their admissions policies. Third, students make their enrollment decisions. My model incorporates tuition, applications, admissions and enrollment as joint outcomes from a subgame perfect Nash equilibrium (SPNE). SPNE in this model need not be unique. Multiplicity may arise

¹In this paper, colleges refer to four-year colleges; students (potential applicants) refer to high school graduates.

²The basic framework of my model builds on the theoretical contribution of Chade, Lewis and Smith (2009).

³The empirical significance of these factors will be assessed in my exercise.

⁴Throughout the paper, student ability refers to her readiness for college.

from two sources: 1) multiple common self-fulfilling expectations held by the student about admissions policies, and 2) the strategic interplay among colleges.⁵

To estimate the model with potentially multiple equilibria, I extend the estimation strategy of Moro (2003) and estimate the model in three steps.⁶ The first two steps recover all the structural parameters involved in the application-admission subgame without having to impose any equilibrium selection rule. In particular, each application-admission equilibrium can be uniquely summarized in the set of probabilities of admission to each college for different types of students. The first step, using simulated maximum likelihood, treats these probabilities as parameters and estimates them along with fundamental student-side parameters in the student decision model, thereby identifying the equilibrium that generated the data. The second step, based on a simulated minimum distance estimation procedure, recovers the college-side parameters by imposing each college's optimal admissions policy. Step three recovers the remaining parameters by matching colleges' optimal tuition with the data tuition.

To implement the empirical analysis, I use data from the National Longitudinal Survey of Youth 1997 (NLSY97), which provides detailed information on student applications, admissions, financial aid and enrollment. Tuition information comes from the Integrated Postsecondary Education Data System.

Some of my major findings are: first, students not only attach different values to the same college, but also rank colleges differently. Second, students face non-trivial application costs that cannot be accounted for simply by application fees. Third, a college's admissions rate, or selectivity, does not necessarily reflect its quality, as measured by the composition of its student body. Among other factors, it strongly depends on the severity of informational frictions and the capacity of the college.⁷

I use the estimated model to conduct three counterfactual experiments. First, I consider the degree to which the market is affected by incomplete information. With a perfect measure of student ability, average student welfare would increase by \$2500, or 6%. All colleges obtain higher-ability students, although their admissions rates increase to almost 100%. This highlights the fact that the selectivity of a college need not reflect its quality in terms

⁵Models with multiple equilibria do not have a unique reduced form and this indeterminacy poses practical estimation problems. In direct maximum likelihood estimation of such models, one should maximize the likelihood not only with respect to the structural parameters but also with respect to the types of equilibria that may have generated the data. The latter is a very complicated task and can make the estimation infeasible.

⁶Moro (2003) estimates a statistical discrimination model in which only one side of the market is strategic. I show how the extended strategy can be used to estimate a model in which both sides of the market are strategic, and hence, the second source of multiple equilibria arises.

⁷In this paper, the capacity of a college refers to the fraction of students it can accommodate.

of enrollee ability.

In the second experiment, I examine the equilibrium impacts of a funding cut to public colleges. All colleges - public and private - increase their tuition. Although the government saves on educational expenses, the loss in student welfare is three times as large as government savings.

In the third experiment, I investigate the extent to which the government can expand college access by increasing the supply of lower-ranked public colleges. At most 2.1% more students can be drawn into colleges, although the enlarged colleges adopt an open admissions policy and lower their tuition to almost zero. Therefore, neither tuition cost nor the number of available slots is a major obstacle to college access. A large group of students, mainly low-ability students, prefer the outside option over any of the college options.

Although this paper is the first to estimate a market equilibrium model that incorporates tuition setting, applications, admissions and enrollment, it builds on various studies on similar topics. For example, Manski and Wise (1983) use nonstructural approaches to study various stages of the college admissions problem separately in a partial equilibrium framework. Most relevant to this paper, they find that applicants do not necessarily prefer the highest quality school.⁸ Arcidiacono (2005) develops and estimates a structural model to address the effects of college admissions and financial aid rules on future earnings. In a dynamic framework, he models student's application, enrollment and choice of college major and links education decisions to future earnings.

While an extensive empirical literature focuses on student decisions, little work has studied the college market in an equilibrium framework. One exception is Epple, Romano and Sieg (2006). In their paper, students differ in family income and ability (perfectly measured by *SAT*) and make a single enrollment decision.⁹ Given its endowment and gross tuition, each college chooses its financial aid and admissions policies to maximize the quality of education provided to its students. Their model provides an equilibrium characterization of colleges' pricing strategies, where colleges with higher endowments enjoy greater market power and provide higher-quality education. With complete information, no uncertainty and no unobserved heterogeneity, their model predicts that students with the same *SAT* and family income would have the same admission, financial aid and enrollment outcomes. To accommodate data variations, the authors assume measurement errors in *SAT* and family income, which are found to be large.¹⁰

⁸Some examples of papers that focus on the role of race in college admissions include Bowen and Bok (1998), Kane (1998) and Light and Strayer (2002).

⁹In their paper, the application decision is not modeled. It is implicitly assumed that either application is not necessary for admission, or all students apply to all colleges. Accordingly, their empirical analysis is based on a sample of first-year college students.

¹⁰The authors note that "the model may not capture some important aspects of admission and pricing."

This paper departs from Epple, Romano and Sieg (2006) in several respects: 1) College application is a non-trivial problem for the student, as is manifested by the popularity of various application guide programs. Student application decisions also differ substantially. For example, over 50% of high school graduates do not apply to college. However, the college market includes not only those who do apply, but all potential college applicants. To evaluate the effects of alternative education policies, it is necessary to understand the application decisions (including non application) made by all students and how these decisions interact with colleges' decisions. This is one focus of my paper. 2) Students differ in their abilities and preferences for colleges, which are unobservable to the econometrician. Accordingly, I allow for such unobserved heterogeneity in my model, which also helps to accommodate data variations. 3) The college market is subject to information friction: colleges can only observe noisy measures of student ability, and they do not observe student preferences. Therefore, I incorporate incomplete information and uncertainty into my model. As the first two structural papers that study college market equilibrium, Epple, Romano and Sieg (2006) and this paper complement one another: the former provides a more comprehensive view on colleges' pricing strategy, while the latter endogenizes student application as part of the equilibrium in a frictional market.

Theoretically, I build on the work by Chade, Lewis and Smith (2009), who model the decentralized matching of students and two colleges. Students, with heterogeneous abilities, make application decisions subject to application costs and noisy evaluations. Colleges compete for better students by setting admissions standards for student signals.¹¹ I extend their framework in the following ways: On the student side, first, students are heterogeneous in their preferences for colleges as well as in their abilities, both of which are unknown to the colleges. Second, I allow for two noisy measures of student ability. One measure, as the signal in Chade, Lewis and Smith (2009), is subjective and its assessment is known only to the college. The other measure is the objective *SAT* score, which is known both to the student and to the colleges she applies to, and may be used strategically by the student in her applications.¹² Third, in addition to the admission uncertainty caused by noisy evaluations, students are subject to post-application shocks. These shocks incorporate new information for the student before she makes her enrollment decision. For example, the amount of financial aid she can obtain is not known with certainty upon application. Moreover, during

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¹¹Nagypál (2004) analyzes a model in which colleges know student types, but students themselves can only learn their type through normally distributed signals.

¹²For example, a low-ability student with a high *SAT* score may apply to top colleges to which she would not otherwise apply; a high-ability student with a low *SAT* score may apply less aggressively than she would otherwise.

the months between application and enrollment, a student may learn more about the colleges and she may also experience unexpected family and/or job prospects.¹³ Empirically, these shocks are necessary to explain "seemingly sub-optimal" behaviors such as an applicant choosing not to attend any college even if admitted. On the college side, I model multiple colleges, who compete against each other via tuition as well as admissions policies.¹⁴

The rest of the paper is organized as follows: Section 2 lays out the model. Section 3 explains the estimation strategy, followed by a brief discussion of identification. Section 4 describes the data. Section 5 presents empirical results, including parameter estimates and model fit. Section 6 describes the counterfactual experiments. The last section concludes the paper. The online appendix contains some details and additional tables.

2 Model

In this section, I will present the model. A discussion about some modeling choices is provided at the end of this section.

2.1 Primitives

2.1.1 Players

There are J colleges, indexed by $j = 1, 2, \dots, J$. In the following, J will also denote the set of colleges. A college's payoff depends on the total expected ability of its enrollees and its tuition revenue. To maximize its payoff, each college has the latitude to choose its tuition and admissions policies, subject to its fixed capacity constraint κ_j , where $\kappa_j > 0$ and $\sum_{j \in J} \kappa_j < 1$.

There is a continuum of students, making college application and enrollment decisions. Students differ in SAT scores and family backgrounds, and they are of different types. In addition, each student also has her own idiosyncratic (permanent) tastes for colleges. SAT scores ($SAT \in \{1, 2, \dots, \overline{SAT}\}$) and family backgrounds (B) are jointly distributed according to $H(SAT, B)$.¹⁵ A student type T is defined as $T \equiv (A, Z) \in \{1, 2, \dots, \overline{A}\} \times \{1, 2, \dots, \overline{Z}\}$, with A denoting ability, and Z representing the non-ability dimension.¹⁶ Type T is correlated with (SAT, B) and distributed according to $P(T|SAT, B) = \Lambda(A|SAT, B)P(Z|A)$, where

¹³For enrollment in the fall semester starting from September, the typical application deadline is in January.

¹⁴As a price of these extensions, it is infeasible to obtain an analytical or graphical characterization of the equilibrium as in Chade, Lewis and Smith (2009).

¹⁵The distribution of $H(SAT, B)$ is nonparametric and comes directly from the data.

¹⁶For example, some students prefer big universities while some prefer small colleges. Type Z captures heterogeneity along these lines.

$\Lambda(A|SAT, B)$ is an ordered logistic distribution and $P(Z|A)$ is non-parametric.¹⁷ Student's idiosyncratic (permanent) tastes for colleges are captured by a J -dimensional random vector ϵ . ϵ is i.i.d. $N(0, \Omega_\epsilon)$, where Ω_ϵ is a diagonal matrix with $\sigma_{\epsilon_j}^2$ denoting the variance of ϵ_j .

2.1.2 Application Cost

Applications are costly to the student. The cost of application is a function of the number of applications sent, regardless of where they are sent. The cost function, denoted as $C(\cdot)$, satisfies the following: $C : \{1, \dots, J\} \rightarrow \mathbb{R}_{++}$, with $C(n+1) \geq C(n)$. I treat $C(\cdot)$ non-parametrically.

2.1.3 Financial Aid

A student may obtain financial aid that helps to fund her attendance in any college, and she may also obtain college-specific financial aid. The amounts of various financial aid depend on the student's family background and SAT , via the exogenous financial aid functions $f_j(B, SAT)$, for $j = 0, 1, \dots, J$, with 0 denoting the general aid and j denoting college j -specific aid. The final realizations are subject to post-application shocks $\eta \in \mathbb{R}^{J+1}$. η is i.i.d. $N(0, \Omega_\eta)$, where Ω_η is a diagonal matrix with $\sigma_{\eta_j}^2$ denoting the variance of shock η_j . The realized financial aid for student i is given by

$$f_{ji} = \max\{f_j(B_i, SAT_i) + \eta_{ji}, 0\} \text{ for } j = 0, 1, \dots, J.$$

2.1.4 Student Preference

Given tuition profile $t \equiv \{t_j\}_{j=1}^J$, the ex-post value of attending college j for student i is given by

$$u_{ji}(t) = (-t_j + f_{0i} + f_{ji}) + (\bar{u}_{jT_i} + \epsilon_{ji}), \quad (1)$$

where t_j is tuition for attending college j . The first parenthesis of (1) summarizes student i 's net monetary cost to attend college j . Her expected total payoff, net of effort cost, is captured by: $\bar{u}_{jT_i}$, type T_i -specific preference, and ϵ_{ji} , her idiosyncratic taste.¹⁸

An outside option is always available to the student and its net expected value is normalized to zero. After application, the outside option is subject to a random shock ζ , which is i.i.d. $N(0, \sigma_\zeta^2)$, and the ex-post value of the outside option is $u_{0i} = \zeta_i$.

¹⁷See the appendix for detailed functional forms.

¹⁸ $\bar{u}_{jT}$'s are treated non-parametrically.

2.1.5 College Payoff

The payoff for college $j \in J$ is given by:

$$\pi_j = \sum_{a=1}^{\bar{A}} \omega_a n_{ja} + (m_{j1}t_j + m_{j2}t_j^2) \times \sum_{a=1}^{\bar{A}} n_{ja}, \quad (2)$$

where ω_a is the value of ability $A = a$, with $\omega_{a+1} > \omega_a$ and ω_1 normalized to 1. n_{ja} is the measure of j 's enrollees with $A = a$. The first term in (2) is college j 's total enrollee ability. The second term in (2) is college j 's payoff from its tuition revenue, where m_j is college j 's valuation of tuition relative to that of enrollee ability.¹⁹

2.1.6 Timing

First, colleges simultaneously announce tuition levels, to which they commit. Second, students make their application decisions, and all colleges simultaneously choose admissions policies. Finally, students learn about admission results and post-application shocks, and then make enrollment decisions.²⁰

2.1.7 Information Structure

Upon student i 's application, each college she applies to receives a signal $s \in \{1, 2, \dots, \bar{s}\}$ drawn from the distribution $P(s|A_i)$, the realization of which is known only to the college. For $A < A'$, $P(s|A')$ first order stochastically dominates $P(s|A)$.²¹ Conditional on the student's ability, signals are i.i.d. across the colleges she applies to.

$P(s|A)$, the distributions of characteristics, preferences, payoff functions and financial aid functions are public information. An individual student's *SAT* score is known both to her and to the colleges she applies to. A student has private information about her type T , her idiosyncratic taste ϵ and her family background B . To ease notation, let $X \equiv (T, B, \epsilon)$. After application, the student observes her post-application shocks. For any individual applicant, college j observes only her *SAT* and the signal she sends to j . In particular, it does not observe whether the student also applies to other colleges.

¹⁹If college j uses tuition only as a tool to maximize enrollee ability, m_j would be 0.

²⁰This paper excludes early admissions, which is a very interesting and important game among top colleges. See, for example, Avery, Fairbanks and Zeckhauser (2003), and Avery and Levin (2010). For college applications in general, however, early admissions account for only a small fraction of the total applications. For example, in 2003, 17.7% of all four-year colleges offered early decision. In these colleges, the mean percentage of all applications received through early decision was 7.6%. *Admission Trends Survey* (2004), National Association for College Admission Counseling.

²¹That is, if $A < A'$, then for any $s \in \{1, 2, \dots, \bar{s}\}$, $\Pr(s' \leq s|A) \geq \Pr(s' \leq s|A')$.

2.2 Applications, Admissions and Enrollment

2.2.1 Enrollment Decision

Knowing her post-application shocks and admission results, student i chooses the best among her outside option and admissions on hand, i.e., $\max\{u_{0i}, \{u_{ji}(t)\}_{j \in O_i}\}$, where O_i denotes the set of colleges that have admitted student i . For students not admitted anywhere, $\max\{u_{ji}(t)\}_{j \in \emptyset} = -\infty$. Let

$$v(O_i, X_i, SAT_i, \eta_i, \zeta_i | t) \equiv \max\{u_{0i}, \{u_{ji}(t)\}_{j \in O_i}\} \quad (3)$$

be the optimal ex-post value for student i , given admission set O_i ; and denote the associated optimal enrollment strategy as $d(O_i, X_i, SAT_i, \eta_i, \zeta_i | t)$.

2.2.2 Application Decision

Given her admissions probability $p_j(A_i, SAT_i | t)$ to each college j , which depends on her ability and SAT , the value of application portfolio $Y \subseteq J$ for student i is

$$V(Y, X_i, SAT_i | t) \equiv \sum_{O \subseteq Y} \Pr(O | A_i, SAT_i, t) E[v(O_i, X_i, SAT_i, \eta_i, \zeta_i | t)] - C(|Y|), \quad (4)$$

where the expectation is over shocks (η_i, ζ_i) , and $|Y|$ is the size of portfolio Y .

$$\Pr(O | A_i, SAT_i, t) = \prod_{j \in O} p_j(A_i, SAT_i | t) \prod_{k \in Y \setminus O} (1 - p_k(A_i, SAT_i | t))$$

is the probability that the set of colleges $O \subseteq Y$ admit student i . The student's application problem is

$$\max_{Y \subseteq J} \{V(Y, X_i, SAT_i | t)\}. \quad (5)$$

Let the optimal application strategy be $Y(X_i, SAT_i | t)$.

2.2.3 Admissions Policy

Given tuition, a college chooses its admissions policy to maximize its expected payoff, subject to its capacity constraint. Its optimal admissions policy must be a best response to other colleges' admissions policies while accounting for students' strategic behavior. In particular, observing only signals and SAT scores of its applicants, the college has to infer: first, the probability that a certain applicant will accept its admission, and second, the expected ability of this applicant conditional on her accepting the admission, both of which depend on the

strategies of all other players.²²

Formally, given tuition profile t , students' strategies $Y(\cdot), d(\cdot)$ and other colleges' admissions policies e_{-j} , college j solves the following problem:

$$\begin{aligned} \max_{e_j(\cdot|t)} \{ & \sum_{s, SAT} e_j(s, SAT|t) \alpha_j(s, SAT|t, e_{-j}, Y, d) \gamma_j(s, SAT|\cdot) \mu_j(s, SAT|\cdot) \\ & + (m_{j1}t_j + m_{j2}t_j^2) \sum_{s, SAT} e_j(s, SAT|t) \alpha_j(s, SAT|\cdot) \mu_j(s, SAT|\cdot) \} \\ \text{s.t. } & \sum_{s, SAT} e_j(s, SAT|t) \alpha_j(s, SAT|t, e_{-j}, Y, d) \mu_j(s, SAT|t, e_{-j}, Y, d) \leq \kappa_j \\ & e_j(s, SAT|t) \in [0, 1], \end{aligned} \quad (6)$$

where $e_j(s, SAT|t)$ is college j 's admissions policy for its applicants with (s, SAT) , $\alpha_j(s, SAT|t, e_{-j}, Y, d)$ is the probability that such an applicant will accept college j 's admission, $\gamma_j(s, SAT|t, e_{-j}, Y, d)$ is the expected ability of such an applicant conditional on her accepting j 's admission, and $\mu_j(s, SAT|t, e_{-j}, Y, d)$ is the measure of j 's applicants with (s, SAT) .²³ The first order condition for problem (6) is

$$\gamma_j(s, SAT|t, e_{-j}, Y, d) + m_{j1}t_j + m_{j2}t_j^2 - \lambda_j + \nu_0 - \nu_1 = 0,$$

where λ_j is the multiplier associated with capacity constraint, i.e., the shadow price of a slot in college j . ν_0 and ν_1 are adjusted multipliers associated with the constraint that $e_j(s, SAT) \in [0, 1]$.²⁴

If it admits an applicant with (s, SAT) and the applicant accepts the admission, college j must surrender a slot from its limited capacity, thus inducing the marginal cost λ_j . The marginal benefit is the expected ability of such an applicant conditional on her accepting j 's admission plus her tuition contribution. Balancing between the marginal benefit and the marginal cost, the solution to college j 's admissions problem is characterized by:

$$e_j(s, SAT|t) \begin{cases} = 1 & \text{if } \gamma_j(s, SAT|t, e_{-j}, Y, d) + m_{j1}t_j + m_{j2}t_j^2 - \lambda_j > 0 \\ = 0 & \text{if } \gamma_j(s, SAT|t, e_{-j}, Y, d) + m_{j1}t_j + m_{j2}t_j^2 - \lambda_j < 0 \\ \in [0, 1] & \text{if } \gamma_j(s, SAT|t, e_{-j}, Y, d) + m_{j1}t_j + m_{j2}t_j^2 - \lambda_j = 0 \end{cases}, \quad (7)$$

²²Conditioning on acceptance is necessary to make a correct inference about the student's ability because of the potential "winner's curse": the student might accept college j 's admission because she is of low ability and is rejected by other colleges.

²³Appendix A.1 provides details on how to calculate $\alpha_j(\cdot)$ and $\gamma_j(\cdot)$.

²⁴ ν_0, ν_1 are the multiplier associated with $\alpha_j(s, SAT|\cdot) \mu_j(s, SAT|\cdot) e_j(s, SAT) \in [0, 1]$.

$$\sum_{s, SAT} e_j(s, SAT|t) \alpha_j(s, SAT|t, e_{-j}, Y, d) \mu_j(s, SAT|t, e_{-j}, Y, d) \leq \kappa_j, \quad (8)$$

and

$$\lambda_j \begin{cases} \geq 0 & \text{if (8) is binding} \\ = 0 & \text{if (8) is not binding} \end{cases}.$$

To implement its admissions policy, college j will first rank its applicants with different (s, SAT) by their $\gamma_j(s, SAT|t, e_{-j}, Y, d)$. All applicants with the same (s, SAT) are identical to the college and hence are treated equally. Everyone in an (s, SAT) group will be admitted if 1) this (s, SAT) group is ranked highest among the groups whose admissions are still to be decided, 2) $\gamma_j(s, SAT|\cdot) + m_{j1}t_j + m_{j2}t_j^2 \geq 0$, and 3) the expected enrollment of this group, $\alpha_j(s, SAT|\cdot)\mu_j(s, SAT|\cdot)$, is no larger than college j 's remaining capacity, where j 's remaining capacity equals κ_j minus the sum of expected enrollment of groups ranked above. A random fraction of an (s, SAT) group is admitted if 1) and 2) hold but 3) fails, where the fraction equals the remaining capacity divided by the expected enrollment of this group. As a result, a typical set of admissions policies for the ranked (s, SAT) groups would be $\{1, \dots, 1, \varepsilon, 0, \dots, 0\}$, with $\varepsilon \in (0, 1)$ if the capacity constraint is binding, and $\{1, \dots, 1\}$ if the capacity constraint is not binding or just binding.

2.2.4 Link Among Various Players

The probability of admission to each college for different (A, SAT) groups of students, $\{p_j(A, SAT|t)\}$, summarizes the link among various players. Knowledge of p makes the information about admissions policies $\{e_j(s, SAT|t)\}$ redundant. Students' application decisions are based on p . College j can make inferences about $\alpha_j(\cdot)$ and $\gamma_j(\cdot)$, and therefore choose its admissions policy, based on p_{-j} . The relationship between p and e is given by:²⁵

$$p_j(A, SAT|t) = \sum_s P(s|A) e_j(s, SAT|t). \quad (9)$$

2.2.5 Application-Admission Equilibrium

Definition 1 *Given tuition profile t , an application-admission equilibrium, denoted as $AE(t)$, is $(d(\cdot|t), Y(\cdot|t), e(\cdot|t))$, such that*

- (a) $d(O, X, SAT, \eta, \zeta|t)$ is an optimal enrollment decision for every (O, X, SAT, η, ζ) ;
- (b) *Given $e(\cdot|t)$, $Y(X, SAT|t)$ is an optimal college application portfolio for every (X, SAT) , i.e., solves problem (5);*

²⁵The role of p as the link among players and the mapping (9) are of great importance in the estimation strategy to be specified later.

(c) For every j , given $(d(\cdot|t), Y(\cdot|t), e_{-j}(\cdot|t))$, $e_j(\cdot|t)$ is optimal admissions policy for college j , i.e., solves problem (6).

2.3 Tuition Policy

Before the application season begins, colleges simultaneously announce their tuition levels, understanding that their announcements are binding and will affect the application-admission subgame. Although from the econometrician's point of view the subsequent game could admit multiple equilibria, I assume that the players agree on the equilibrium selection rule.²⁶ Let $E(\pi_j|AE(t))$ be college j 's expected payoff under $AE(t)$. Given t_{-j} and the equilibrium profiles $AE(\cdot)$ in the following subgame, college j 's problem is

$$\max_{t'_j \geq 0} \{E(\pi_j|AE(t'_j, t_{-j}))\}. \quad (10)$$

Independent of m , each college considers the strategic role of its tuition in the subsequent $AE(t'_j, t_{-j})$. On the one hand, low tuition makes the college more attractive to students and more competitive in the market. On the other hand, high tuition serves as a screening tool and leads to a better pool of applicants if high-ability students are less sensitive to tuition than low-ability students.²⁷ Together with the monetary incentives of tuition revenue, such trade-offs determine the college's optimal tuition level.

2.4 Subgame Perfect Nash Equilibrium

Definition 2 A subgame perfect Nash equilibrium for the college market is

$(t^*, d(\cdot|t^*), Y(\cdot|t^*), e(\cdot|t^*))$ such that:

- (a) For every t , $(d(\cdot|t), Y(\cdot|t), e(\cdot|t))$ constitutes an $AE(t)$, according to Definition 1;
- (b) For every j , given t_{-j}^* , t_j^* is optimal tuition for college j , i.e., solves problem (10).

In the online appendix, I show the existence of equilibrium for a simplified version of the model with two colleges. The logic of the proof should apply to the full model. However, a formal proof poses a great challenge, one which I have not yet been able to conquer. Numerically, I have found equilibrium in all cases throughout my empirical analyses.

²⁶The way in which the equilibrium selection rule is reached is beyond the scope of this paper. But as an example, it may result from repeated interactions between players.

²⁷This is a possible scenario. However, in the estimation, I do not impose any restriction on the relationship between student ability and their sensitivity to prices.

2.5 Discussion: A Closer Look at Modeling Choices

2.5.1 Financial Aid

In the model, financial aid is an exogenous stochastic function of student characteristics. Arguably, financial aid is an important tool for colleges to compete for better students. Epple, Romano and Sieg (2006) follow in this direction, however, they abstract from application decisions to keep their analysis feasible. Modeling strategic financial aid decisions together with student application decisions involves establishing a fixed point of financial aid to establish an equilibrium. However, financial aid, as a function of student characteristics, is of infinite dimension. Treating it as an equilibrium object together with applications makes the empirical analysis intractable. I carry out my analysis in a way that complements Epple, Romano and Sieg (2006), leaving the development of a modeling framework that combines the strengths of these two papers to future research.

2.5.2 Student Preference

I treat students' type-specific preferences for colleges as parameters. These preferences reflect the expected total payoffs (net of effort cost) from attending a certain college group, which differ across student types. One of the most important factors underlying these preferences is students' job market outcomes after college.²⁸ I am unable to model these underlying forces primarily due to data limitations: most students in my sample are still in college and I do not observe their job market outcomes.²⁹ With limited information, I choose to impose minimum structure on these preferences rather than making assumptions about how they are formed. As future surveys from the NLSY97 become available, it will be promising to model these underlying forces.

Another potentially important component is peer quality.³⁰ In the current model, given admissions probabilities, students' application strategies are independent, which yields a unique equilibrium in the student-side problem. With peer effects, multiple equilibria may coexist in both the student-side and the college-side problem, inducing substantial complications into the model. In this paper, I focus on the interactions between colleges and students and the competition among colleges.

²⁸See, for example, Arcidiacono (2005).

²⁹Most college applications in the sample occurred in 2003. Given the panel structure of NLSY 97, this extension is feasible as future survey data become available.

³⁰The literature on peer effects in higher education has been controversial. For example, Sacerdote (2001) and Zimmerman (2003) find peer effects between roommates on grade point averages. Arcidiacono and Nicholson (2005) find no peer effects among medical students. Dale and Krueger (1998) have mixed findings.

2.5.3 College Objective

In this model, I assume that colleges value student ability and tuition revenue.³¹ For example, higher-ability students are more likely to do better in the job market, which enhances the college's prestige and attractiveness to future applicants. To incorporate such a mechanism requires a long-run dynamic model. Without data necessary for analyzing the long-run equilibrium, I focus on the short-run equilibrium. For the same reason, I assume that colleges are faced with exogenous capacity constraints, which could be another control variable in the long run.

I assume that colleges' preferences for student ability and tuition revenue are separable. However, since tuition affects a college's pool of applicants, it enters the college's objective function in a non-separable way. A non-linear preference over tuition is assumed because most colleges are non-profit, and a non-linear preference allows for possibly satiated preference over tuition.³²

2.5.4 Information Structure

I assume that a student knows her own ability whereas colleges can only observe noisy measures of student ability. One might argue that neither party knows student ability. However, I believe it is reasonable to assume that students have information advantages relative to the colleges to which they are applying. As high school graduates, students have been evaluated repeatedly. Although these evaluations may be noisy, students eventually learn their ability by observing these signals over time. It is feasible and interesting to extend the current model to a case where both parties are uncertain about student ability. In this paper, I focus on the special case, which captures the main idea of information asymmetry of the type that I consider.

3 Estimation Strategy and Identification

3.1 Estimating Application-Admission Subgame

First, I fix the tuition profile at its equilibrium (data) level and estimate the parameters that govern the application-admission subgame. To save notation, I suppress the dependence of endogenous objects on tuition.

³¹A detailed modeling of how colleges set up their objectives is important but is beyond the scope of this paper.

³²I also tried some other functional forms. For example, a linear preference over tuition predicts much higher tuition levels than are observed in the data.

The estimation is complicated by potential multiple equilibria in the subgame and that econometricians do not observe the equilibrium selection rule.³³ One way to deal with this complication is to impose some equilibrium selection rule assumed to have been used by the players and to consider only the selected equilibrium. However, for models like the one in this paper, there is not a single compelling selection rule (from the econometrician's point of view).³⁴ I use a two-step strategy to estimate the application-admission subgame without having to impose any equilibrium selection rule.

Each application-admission equilibrium is uniquely summarized in the admissions probabilities $\{p_j(A, SAT)\}$, which provide sufficient information for players to make their unique optimal decisions. In the student decision model, $\{p_j(A, SAT)\}$ are taken as given. Step One treats $\{p_j(A, SAT)\}$ as parameters and estimates them along with structural student-side parameters, thereby identifying the equilibrium that generated the data. Step two imposes colleges' optimal admissions policies, which yield a new set of admissions probabilities. Under the true college-side parameters, these probabilities should match the equilibrium admissions probabilities estimated in the first step.

3.1.1 Step One: Estimate Fundamental Student-Side Parameters and Equilibrium Admissions Probabilities

I implement the first step via simulated maximum likelihood estimation (SMLE): together with estimates of the fundamental student-side parameters $(\hat{\Theta}_0)$, the estimated equilibrium admissions probabilities $\hat{p}$ should maximize the probability of the observed outcomes of applications, admissions, financial aid and enrollment, conditional on observable student characteristics, i.e., $\{(Y_i, O_i, f_i, d_i | SAT_i, B_i)\}_i$. Θ_0 is composed of 1) type-specific preference parameters and idiosyncratic taste distribution parameters $\Theta_{0u} = [\{\bar{u}_j(T)\}, \{\sigma_{\epsilon_j}\}]'$, 2) application cost parameters $\Theta_{0C} = \{C(n)\}'$, 3) financial aid parameters Θ_{0f} , 4) the standard deviation of the shock to the outside option $\Theta_{0\zeta} = \sigma_\zeta$ and 5) the parameters involved in the distribution of types Θ_{0T} .

Suppose student i is of type T . Her contribution to the likelihood, denoted by

$L_{iT}(\Theta_{0u}, \Theta_{0C}, \Theta_{0f}, \Theta_{0\zeta}, p)$, is composed of the following parts:

$L_{iT}^Y(\Theta_{0u}, \Theta_{0C}, \Theta_{0f}, \Theta_{0\zeta}, p)$ — the contribution of Y_i ,

$L_{iT}^O(p)$ — the contribution of $O_i | Y_i$,

$L_{iT}^f(\Theta_{0f})$ — the contribution of $f_i | O_i$, and

³³The problem of possible multiple equilibria is a difficult, yet frequent problem in structural equilibrium models. For example, the model by Epple, Romano and Sieg (2006) also admits multiple equilibria, and the authors assume unique equilibrium in their estimation and other empirical analysis.

³⁴See, for example, Mailath, Okuno-Fujiwara and Postlewaite (1993), who question the logical foundations and performances of many popular equilibrium selection rules.

$L_{iT}^d(\Theta_{0u}, \Theta_{0f}, \Theta_{0\zeta})$ — the contribution of $d_i|(O_i, f_i)$.

Hence,

$$L_{iT}(\cdot) = L_{iT}^Y(\cdot) L_{iT}^O(\cdot) L_{iT}^f(\cdot) L_{iT}^d(\cdot).$$

Now, I will specify each part in detail. Conditional on (T, SAT_i, B_i) , there are no unobservables involved in the probabilities of $O_i|Y_i$ and $f_i|O_i$. The probability of $O_i|Y_i$ depends only on ability and SAT , and is given by

$$L_{iT}^O(p) \equiv \Pr(O_i|Y_i, A, SAT_i) = \prod_{j \in O_i} p_j(A, SAT_i) \prod_{k \in Y_i \setminus O_i} [1 - p_k(A, SAT_i)].$$

Let $J_i^f \subseteq \{0, O_i\}$ be the sources of observed financial aid for student i , where 0 denotes general aid. The probability of the observed financial aid depends only on SAT and family background:

$$\begin{aligned} L_{iT}^f(\Theta_{0f}) &\equiv \Pr(f_i|O_i, SAT_i, B_i) \\ &= \begin{cases} \prod_{j \in J_i^f} \phi\left(\frac{f_{ji} - f_j(SAT_i, B_i)}{\sigma_{\eta_j}}\right) I(f_{ji} > 0) \Phi\left(\frac{-f_j(SAT_i, B_i)}{\sigma_{\eta_j}}\right) I(f_{ji} = 0) & \text{if } J_i^f \neq \emptyset \\ 1 & \text{otherwise} \end{cases}, \end{aligned}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the standard normal density and cumulative distribution, respectively, and $I(\cdot)$ is the indicator function.

The choices of Y_i and $d_i|(O_i, f_i)$ both depend on the unobserved idiosyncratic tastes ϵ . Let $G(\epsilon, \zeta, \{\eta_j\}_{j \in \{0, O_i\} \setminus J_i^f})$ be the joint distribution of idiosyncratic taste, outside option shock and unobserved financial aid shocks,

$$\begin{aligned} L_{iT}^Y(\Theta_{0u}, \Theta_{0C}, \Theta_{0f}, \Theta_{0\zeta}, p) L_{iT}^d(\Theta_{0u}, \Theta_{0f}, \Theta_{0\zeta}) &\equiv \\ \int I(Y_i|T, SAT_i, B_i, \epsilon) I(d_i|O_i, T, SAT_i, B_i, \epsilon, \zeta, \{\eta_j\}_{j \in \{0, O_i\} \setminus J_i^f}, \{f_{ji}\}_{j \in J_i^f}) & \\ dG(\epsilon, \zeta, \{\eta_j\}_{j \in \{0, O_i\} \setminus J_i^f}). & \end{aligned}$$

The multi-dimensional integration has no closed-form solution and is approximated by a kernel smoothed frequency simulator (McFadden (1989)).³⁵

To obtain the likelihood contribution of student i , I integrate over the unobserved type:

$$L_i(\Theta_0, p) = \sum_T P(T|SAT_i, B_i; \Theta_{0T}) L_{iT}(\Theta_{0u}, \Theta_{0C}, \Theta_{0f}, \Theta_{0\zeta}, p). \quad (11)$$

³⁵ See the online appendix for details.

Finally, the log likelihood for the entire random sample is

$$\tilde{L}(\Theta_0, p) = \sum_i \ln(L_i(\Theta_0, p)). \quad (12)$$

3.1.2 Step Two: Estimate College-Side Parameters

The college-side parameters to be estimated in Step Two, denoted Θ_2 , are signal distribution $P(s|A)$, capacity constraints κ and values of abilities ω . These are estimated via simulated minimum distance estimation (SMDE). Based on $\hat{\Theta}_0$, I simulate a population of students and obtain their optimal application and enrollment strategies under $\hat{p}$. The resulting equilibrium enrollment in each college group should equal its expected capacity.³⁶ These equilibrium enrollments, together with $\hat{p}$, serve as targets to be matched in the second-step estimation.

The estimation explores each college's optimal admissions policy: taking student strategies and $\hat{p}_{-j}$ as given, college j chooses its admissions policy e_j . This leads to the admissions probability to college j for each (A, SAT) type, according to equation (9). Ideally, the admissions probabilities derived from Step Two should match $\hat{p}$ from Step One, and the capacity parameters in Step Two should match equilibrium enrollments. The estimates of the college-side parameters minimize the weighted sum of the discrepancies. Let $\hat{\Theta}_1 = [\hat{\Theta}'_0, \hat{p}']'$; the objective function in Step Two is

$$\min_{\Theta_2} \{q(\hat{\Theta}_1, \Theta_2)' \widehat{W} q(\hat{\Theta}_1, \Theta_2)\}, \quad (13)$$

where $q(\cdot)$ is the vector of the discrepancies mentioned above, and $\widehat{W}$ is an estimate of the optimal weighting matrix. The choice of W takes into account that $q(\cdot)$ is a function of $\hat{\Theta}_1$, which are point estimates with variances and covariances.

³⁶It is implicitly assumed that the tuition weights m are such that, at the data tuition level, the marginal benefit from admitting a student is non-negative, i.e., $\gamma_j(s, SAT|\cdot) + m_{j1}t_j + m_{j2}t_j^2 \geq 0$ for any (s, SAT) . Given that $\gamma_j(s, SAT|\cdot) \geq 1$ by definition, this assumption means that the college does not "dislike" tuition too much. Under this assumption, capacity constraints are binding in the realized equilibrium because the data admissions rates are below 100% for all college groups.

3.2 Tuition Weights

Given other colleges' equilibrium (data) tuition t_{-j}^* , I solve college j 's tuition problem (10).³⁷ Under the true tuition weight parameters m , the optimal solution should match the data tuition.³⁸ The objective in Step Three is

$$\min_m \{(t^* - t(\hat{\Theta}, m))'(t^* - t(\hat{\Theta}, m))\},$$

where t^* is the data tuition profile, $t(\cdot)$ consists of each college's optimal tuition, and $\hat{\Theta} \equiv [\hat{\Theta}_0, \hat{\Theta}_2]$ is the vector of fundamental parameter estimates from the previous two steps. I obtain the variance-covariance of $\hat{m}$ using the Delta method, which exploits the variance-covariance structure of $\hat{\Theta}$.

3.3 Identification

3.3.1 Student-Side Parameters

The student-side model can be viewed as a finite mixture of multinomial probits. In the online appendix, I prove the identification of a mixed probit model with two types.³⁹ The identification in the more general case of mixed multinomial probits with multiple types would require more complicated algebraic analysis but would nevertheless follow the same logic. The observed variation in students' behavior arises from their heterogeneity both across and within types. In order to disentangle these two sources of heterogeneity, I need additional within-type variation that is driven by some observables. I assume that only *SAT* and family income (a 5-year average) enter the type distribution, i.e., *SAT* and family (permanent) income summarize all information that correlates with ability. By contrast, financial aid

³⁷ Given $\hat{\Theta}$, t_{-j}^* and some m , I examine college j 's expected payoff at each trial tuition level t_j' and obtain the optimal tuition associated with this m . This procedure requires computing the series of application-admission equilibria $AE(\cdot, t_{-j}^*)$, which can only be achieved through simulation. To do so, I use an algorithm motivated by the rule of "continuity of equilibria," which requires, intuitively, that $AE(t_j', t_{-j}^*)$ be close to $AE(t_j, t_{-j}^*)$ when t_j' is close to t_j . Specifically, I start from the equilibrium at the data tuition level (t_j^*, t_{-j}^*) , which is numerically unique for nontrivial initial beliefs ($p \gg 0$). $AE(t^*)$ is found to be unique numerically in my search for equilibrium starting from 500 different combinations of nontrivial initial beliefs.

³⁸ Given that there is only a single college market, there are only four tuition observations on which to base the estimation of the colleges' objective functions. Therefore, pursuing a conventional estimation approach is not sensible. Instead, I treat the four nonlinear best response functions as exact, which implies that the econometrician observes all factors involved in a college's tuition decision, and saturate the model. This approach also enables me to recover the tuition weights without solving the full equilibrium of the model. As is shown below, the fit to the tuition data is quite good, although there is no statistical criterion that can be applied.

³⁹ The proof builds on Meijer and Ypma (2008), who show the identification for a mixture of two univariate normal distributions.

depends on SAT and all family-background variables.⁴⁰ For example, conditional on family permanent income, family assets vary with factors, such as housing prices and stock prices, that are not correlated with ability. Variations in financial aid have different impacts on students across types, which helps to identify the type distribution and type-specific utilities.

Given the type distribution identified from the mixture of probits, I now discuss the major sources for the identification of other student-side parameters. First, the probabilities of admissions $\{p_j(A, SAT)\}$ are identified mainly from the observed variation in admissions across students with the same SAT but different family income, due to the exclusion restriction that family income affects admissions rates only via ability. Second, the standard deviation of the i.i.d. idiosyncratic tastes σ_ϵ is identified from the variation in expected financial aid across students within a college, given that student utility is measured in monetary units and that the coefficient on net tuition is normalized to one. Third, application costs are identified mainly from the variation in the numbers of applications, using the restriction that $C(\cdot)$ is the same across students.⁴¹ Finally, the fraction of admitted students who chose not to attend any college serves as the major identification source for σ_ζ , the standard deviation of the outside option shock.

3.3.2 College-Side Parameters

The identification of capacity parameters κ follows directly from the equilibrium college enrollments calculated based on $\hat{\Theta}_1$. The identification of $\{\hat{P}(s|A)\}$ is facilitated by the restriction that signal distribution is the same across colleges. However, the vector of ability values ω is not point identified, even after normalizing ω_1 . The reasoning is as follows: each college j faces discrete (s, SAT) groups of applicants and its admissions policy depends on the rankings of these groups in terms of their conditional expected abilities. These relative rankings remain unchanged for a range of ω 's, implying that ω cannot be point identified. Consequently, I set up a grid of ω 's and implement the second step estimation given each of these ω 's. The best fit occurs with ω 's around $[1, 2, 3]'$; therefore, I fix $\hat{\omega} = [1, 2, 3]'$. At other values of ω around $[1, 2, 3]'$, the estimates for the other parameters in steps two and three will change accordingly. However, the counterfactual experiment results are robust.⁴²

⁴⁰This exclusion restriction is sufficient but not necessary for identification. For example, I could allow family assets to enter type distribution as a categorical variable, and to enter the financial aid function as a continuous variable. The within-category variation in assets would be enough for identification.

⁴¹For example, by comparing $V(\{j\})$ and $V(\emptyset)$, I can identify $\left(\bar{u}_j(A, Z) - \frac{C(1)}{p_j(SAT, A)}\right)$. Then I can separately identify $\bar{u}_j$ and $C(1)$ because application cost is independent of SAT .

⁴²The online appendix shows the counterfactual experiment results with alternative ω 's around $[1, 2, 3]'$.

4 Data

4.1 NLSY Data and Sample Selection

In NLSY97, a college choice series was administered in years 2003-2005 to respondents from the 1983 and 1984 birth cohorts who had completed either the 12th grade or a GED at the time of interview.⁴³ Respondents provided information about each college to which they applied, including name and location; any general financial aid they may have received; whether each college to which they applied had accepted them for admission, along with financial aid offered. Information was asked about each application cycle.⁴⁴ In every survey year, the respondents also reported on the college(s), if any, they attended during the previous year.⁴⁵ Other available information relevant to this paper includes *SAT/ACT* score and financial-aid-relevant family information (family income, family assets, race and number of siblings in college at the time of application).

The sample I use is from the 2303 students within the representative random sample who were eligible for the college choice survey in at least one of the years 2003-2005. To focus on first-time college application behavior, I define applicants as students whose first-time college application occurred no later than 12 months after they became eligible. Under this definition, 1756 students are either applicants or non-applicants.⁴⁶ I exclude applications for early admission.⁴⁷ I also drop observations where some critical information, such as the identity of the college applied to, is missing. The final sample size is 1646.

4.2 Aggregation of Colleges

Two major constraints make it necessary to aggregate colleges. One is computational feasibility: with a large number of colleges, solving the student optimal portfolio problem and/or computing the equilibrium poses major computational challenges.⁴⁸ Another major constraint is sample size: without some aggregation, the number of observations for each option would be too small to obtain precise parameter estimates. Consequently, I aggregate colleges

⁴³See data appendix for more information about NLSY97.

⁴⁴An application cycle includes applications submitted for the same start date, such as fall 2004.

⁴⁵The NLSY97 geocode (restricted-use) data provide information on the names and locations of the colleges to which the student applied.

⁴⁶I exclude students who were already in college before their first reported applications. If a student is observed in more than one cycle, I use only her/his first-time application/non-application information.

⁴⁷There is no direct information on early admission; I identify early admission according to the rules specified in the data appendix.

⁴⁸The choice set for the student application problem grows exponentially with the number of colleges. Moreover, a fixed point has to be found for each college's admissions policy in order to solve for the equilibrium.

into groups and treat each group as one college in the estimation. By doing so, I abstract from some idiosyncratic factors such as regional preferences that may be important at a disaggregate level but are less likely to be important at a more aggregate level.⁴⁹

The aggregation goes as follows: first, I divide all four-year colleges into private and public categories, and then I use the within-category rankings from U.S. News and World Report 2003-2006 for further division.⁵⁰ Table 1 shows the detailed grouping: I group the top 30 private universities and top 20 liberal arts colleges into Group 1, the top 30 public universities into Group 2, and all other four-year private (public) colleges into Group 3 (Group 4).

Table 1.1 Aggregation of Colleges

Variable	Group 1	Group 2	Group 3	Group 4
Num. of colleges (Potential ^a)	51	32	1921	619
Num. of colleges (Applied ^b)	37	32	312	292
Capacity ^c (%)	1.0	4.6	11.2	24.4

Group 1: Top private colleges; Group 2: Top public colleges;

Group 3: Other private colleges; Group 4: Other public colleges.

a. Total number of colleges in each group (IPEDS).

b. Number of colleges applied to by some students in the sample.

c. Capacity = Num. of students in the sample enrolled in group j/sample size.

To accommodate the aggregation of colleges, the empirical definitions of application, admission and enrollment in this paper are as follows: a student is said to have applied to group j if she applied to any college within group j ; is said to have been admitted to group j if she was admitted to any college in group j ; and is said to have enrolled in group j if she enrolled in any college in group j . With these definitions, this paper is meant to capture the behavior of the majority of students: 60% of applicants in the sample applied to no more than one college within a group; on the other hand, cross-group application is a significant phenomenon in the data. Table 1.2 shows, conditional on applying to the college group in the row, the fraction of applicants that applied to each of the college groups in the column. For example, 32.7% of Group 1 applicants also applied to Group 2. Moreover, among the applicants who applied to both groups within the public/private category, very few applied to cross-group colleges that are close in ranking.⁵¹

⁴⁹Epple, Romano and Sieg (2006) focuses on private four-year colleges and aggregates them into six groups.

⁵⁰The report years I use correspond to the years when most of the students in my sample applied to colleges, and the rankings had been very stable during that period. See data appendix for more information.

⁵¹Among the applicants who applied to both groups within the public/private category, I define a student as a "close applicant" if the ranking distance is less than 10 between the best lower-ranked college and the

Table 1.2 Applications|Applied to a Certain Group

%	Group 1	Group 2	Group 3	Group 4
Group 1	100.0	32.7	70.9	40.0
Group 2	12.2	100.0	39.9	52.7
Group 3	13.0	19.6	100.0	47.2
Group 4	4.1	14.5	26.4	100.0

Conditional on applying to the group in the row,
the fraction that applied to each group in the column.

I also adjust the definitions of tuition and financial aid to college aggregation. I use the within-group average tuition as the group tuition, based on the tuition information from the Integrated Postsecondary Education Data System (IPEDS). If a student received financial aid offers from more than one college within the group in which she enrolled, the financial aid from the attended college is viewed as the aid she received from this group; if she was offered aid from more than one college within a group in which she did not enroll, the highest financial aid from that group is used.⁵²

4.3 Summary Statistics

Table 2 summarizes characteristics among non-applicants, applicants and attendees. Clear differences emerge between non-applicants and applicants: the latter are much more likely to be female, white, with higher *SAT* scores and with higher family income. Conditional on applying, attendees and non-attendees do not significantly differ. Similar patterns have been found in other studies using different data.⁵³

Table 3 summarizes the distribution of application portfolio size. Fifty-five percent of students did not apply to any four-year college. Among applicants, 67% applied to only one group, and only 7% of applicants applied to three groups or more. Relating portfolio size to student characteristics: whites, students with higher *SAT* and students with higher family income are more likely to apply to more groups.

worst top college she applied to. Among Group 1-and-Group 3 applicants, 10% are close applicants. Among Group 2-and-Group 4 applicants, there are no close applicants.

⁵²Given the assumption that all colleges are identical within a group, the highest financial aid from the group together with the non-pecuniary utility from that group is the highest bid for the student from that group.

⁵³For example, Arcidiacono (2005), using data from the National Longitudinal Study of the Class of 1972, and Howell (2010), using data from National Education Longitudinal Study of 1988 report similar patterns.

Table 2 Student Characteristics

Variable	Non-Applicants	Applicants	Attendees
Female	43.2%	53.0%	54.1%
Black	17.7%	13.3%	12.1%
Family Income ^a	39835.5 (32361.0)	68481.1 (51337.0)	70605.61 (51279.3)
$Inc^b = 1$	34.5%	13.7%	12.7%
$Inc = 2$	50.8%	48.9%	47.6%
$Inc = 3$	14.7%	37.5%	39.7%
$SAT^c = 1$	79.8%	16.5%	13.7%
$SAT = 2$	17.0%	59.7%	60.6%
$SAT = 3$	3.2%	23.8%	25.7%
Observations	899	747	678

a. in 2003 dollars

b. $Inc=1$ if family income is below 25th percentile (group mean \$10,017)

$Inc=2$ if family income is in 25-75th percentile (group mean \$45,611)

$Inc=3$ if family income is above 75th percentile (group mean \$110,068)

c. $SAT=1$ if SAT or ACT equivalent is lower than 800.^{54,55}

$SAT=2$ if SAT or ACT equivalent is between 800 and 1200.

$SAT=3$ if SAT or ACT equivalent is above 1200.

Table 3 Distribution of Portfolio Sizes (%)

	Size= 0	Size= 1	Size= 2	Size= 3	Size= 4
All	54.6	31.0	11.2	2.9	0.3
White	53.3	31.6	11.7	3.1	0.3
Black	61.6	27.9	8.1	1.9	0.3
$SAT = 1$	85.4	12.7	1.4	0.5	0.0
$SAT = 2$	25.5	50.0	19.0	5.0	0.3
$SAT = 3$	14.0	49.8	28.0	6.8	1.4
$Inc = 1$	75.2	19.7	3.6	1.0	0.5
$Inc = 2$	55.6	32.4	10.3	1.7	0.0
$Inc = 3$	32.0	39.6	20.4	7.3	0.7

Table 4 shows group-specific application rates and admissions rates. The application rate, defined as the fraction of applicants that apply to a certain group, increases as one goes from

⁵⁴Students who did not take the SAT or ACT test are categorized into $SAT=1$ group, since their behavior is very similar to those with low SAT/ACT scores.

⁵⁵Score conversion follows *SAT-ACT* Concordance Tables (College Board).

Group 1 to Group 4.⁵⁶ However relative to their capacities (shown in Table 1.1), top colleges receive disproportionately higher fractions of applications than lower-ranked colleges. For example, Group 4 is almost 25 times as large as Group 1, but the application rate for Group 4 is only 10 times as high as that for Group 1. Consistently, the admissions rate increases monotonically from 58% in Group 1 to 96% in Group 4.

Table 4 Application & Admission: All Applicants (%)

	Group 1	Group 2	Group 3	Group 4
Application Rate	7.4	19.8	40.3	72.0
Admission Rate	58.2	76.4	91.7	95.7

Num of all applicants: 747

Application rate=num. of group j applicants/num. of all applicants

Admission rate=num. of students admitted to group j/num. of group j applicants

Table 5 shows the final distribution of students who obtained at least one admission. Over 80% of them attended lower-ranked colleges, with Group 4 accommodating 56%. Only 2% attended colleges in the top-ranked private Group 1. Six percent of admitted students chose the outside option, suggesting the existence of post-application shocks.

Table 5 Final Allocation of Admitted Students (%)

Group 1	Group 2	Group 3	Group 4	Outside
2.2	10.6	25.6	55.7	6.0

Num. of students with at least one admission: 720.

Table 6 summarizes tuition and financial aid. Private colleges are four to five times as costly as public colleges of similar ranking. Within the public/private category, the higher-ranked colleges are more costly. Relative to students admitted to top groups, a higher fraction of students admitted to lower-ranked groups receive college financial aid. Conditional on obtaining some aid, the amount of aid is monotone in the tuition cost.⁵⁷ As shown in the last column, 40% of admitted students receive some outside financial aid that helps to fund college attendance in general, but the average amount of general aid is lower than that of any college-specific aid.

⁵⁶Application rates across groups will not necessarily add up to 100%, since some students applied to multiple college groups.

⁵⁷Financial aid can exceed tuition, since it may also cover other expenditures necessary for college attendance.

Table 6 Tuition and Financial Aid

	Group 1	Group 2	Group 3	Group 4	General
Tuition ^a	27009	5347	17201	3912	<i>N/A</i>
Fraction of Aid Recipients ^b	42.4%	32.8%	67.1%	46.6%	39.9%
Mean Aid for Recipients	12836.1	8967.9	11346.6	5344.8	4325.6

a. Tuition and aid are measured in 2003 dollars.

b. Num. of aid recipients in the sample/num. of admitted students in the sample

5 Empirical Results

This section presents structural parameter estimates (with standard deviation in parenthesis) and model fits. I allow for six types of students, with $(A, Z) \in \{1, 2, 3\} \times \{1, 2\}$, three *SAT* levels, and three signal levels.⁵⁸ I allow the tuition weight vector m to differ across public and private categories, but restrict it to be the same within the public/private category.

5.1 Student-Side Parameter Estimates

5.1.1 Preference Parameter Estimates

Table 7.1 reports the estimates of preference parameters. Rows 1 to 3 show the mean values attached to colleges by type $Z = 1$ students with $A = 1$ to $A = 3$, respectively. $\bar{\psi}_j(A)$'s shown in the next two rows are the additional values attached to each college group by type $Z = 2$ students relative to type $Z = 1$ students, conditional on ability. That is, $\bar{u}_j(A, Z = 2) = \bar{u}_j(A, Z = 1) + \bar{\psi}_j(A)$.⁵⁹ The next three rows report $\bar{u}_j(A, Z = 2)$. Within the same Z type, students of different ability levels have very different valuations of colleges. An average student of the lowest ability ($A = 1$) derives large negative utility from any college, and her college utility levels are universally much lower than those of higher-ability students. For students of the two higher ability levels, their valuations of colleges are not universally monotone in ability: on average, $A = 3$ students value top colleges (Groups 1,2) more and lower-ranked colleges (Groups 3,4) less than $A = 2$ students do. Since these preference parameters reflect the expected total benefits, net of effort costs, of attending colleges, it is not completely surprising to see non-monotone preferences. It is reasonable to believe that the effort costs required in top colleges are higher than those required in lower-ranked colleges, and that these costs decrease with student ability. Considering the effort costs and the probabilities of success in different colleges, a mediocre student might

⁵⁸ A , *SAT* and s go from low to high as the levels go from 1 to 3.

⁵⁹ I restrict $\bar{\psi}_j(1) = \bar{\psi}_j(2)$.

be better off attending a lower-ranked college. Holding ability constant, $Z = 2$ type value private colleges more and public colleges less than $Z = 1$ type. Private colleges and public colleges have different features that may fit some students better than others. For example, private colleges are usually smaller than public colleges, which may be an advantage for some students but a disadvantage for others.

Table 7.1 Preference Parameter Estimates

	Group 1	Group 2	Group 3	Group 4
$\bar{u}_j(A = 1, Z = 1)$	-233937.51 (79801.99)	-287044.58 (18949.38)	-216980.82 (8143.23)	-119861.20 (4483.13)
$\bar{u}_j(A = 2, Z = 1)$	-222379.95 (43606.00)	-97657.36 (9341.44)	-20898.31 (3203.74)	81493.20 (1096.97)
$\bar{u}_j(A = 3, Z = 1)$	-57506.06 (3536.88)	59719.85 (6397.90)	-52037.77 (6164.44)	11025.54 (4629.37)
$\bar{\psi}_j(A = 1, 2)$	159977.95 (40945.39)	-22777.16 (10115.50)	155710.58 (4188.71)	-124851.87 (6790.05)
$\bar{\psi}_j(A = 3)$	181694.72 (26718.71)	-66612.01 (7528.33)	89018.65 (9597.78)	-115946.48 (21069.53)
$\bar{u}_j(A = 1, Z = 2)$	-73959.56	-309821.73	-61270.24	-244713.07
$\bar{u}_j(A = 2, Z = 2)$	-62402.00	-120434.52	134812.27	-43358.67
$\bar{u}_j(A = 3, Z = 2)$	124188.65	-6892.15	36980.88	-104920.94
σ_{ϵ_j}	115039.64 (1164.68)	91646.92 (3807.92)	77914.55 (1964.19)	43567.67 (1506.58)
σ_ζ		10433.37 (2916.06)		
$\bar{u}_j(A, Z = 2) = \bar{u}_j(A, Z = 1) + \bar{\psi}_j(A)$				

The next row of Table 7.1 shows the standard deviations of idiosyncratic tastes: even within T type, students are still very different in their tastes for colleges. For example, although Group 1 colleges are worth only \$124,188 for an average student of $(A = 3, Z = 2)$ type, this value becomes \$271,618 at the 90th percentile.⁶⁰ The last row shows the estimate of the standard deviation of the ex post shock to the outside option. Relative to the variation in permanent tastes, the variation in the ex post shocks is small: the major driving force in a student's decision is her permanent taste. However, together with the ex post financial aid shocks, the ex post shock to the outside option introduces non-trivial uncertainty into

⁶⁰Table 7.2 in the online appendix illustrates the importance of within-type taste dispersion by showing the mean evaluations of colleges among all students, applicants and attendees, from a simulated example.

a student's application problem. For example, ex post, a student might opt out even with some admissions in hand.

In sum, there is significant heterogeneity in students' preferences for colleges, both across types and within each type. Not only do students attach different values to the same college, but they also rank colleges differently. For example, attending an elite college is not optimal for all students.⁶¹ Instead, each option (including the outside option) offered in the college market best caters to some groups of students.

5.1.2 Type Distribution Parameter Estimates

Table 8.1 shows the parameter estimates for the ordered logit distribution of ability conditional on family income and *SAT*. Students with higher *SAT* scores and those with higher family income are more likely to be of higher ability. Table 8.2 shows the distribution of *Z* types by ability: at all ability levels, most students are of type $Z = 1$ (78% of all students), but the fraction decreases as ability increases. In other words, higher-ability students are more likely to be of the type that values private colleges over public colleges.

Table 8.1 Ordered Logit Ability Distribution

cut_1^a	cut_2^a	Family Income	$SAT = 2$	$SAT = 3$
2.4782	5.4100	0.00001	2.8052	3.6927
(0.1555)	(0.2220)	(0.000002)	(0.16147)	(0.2297)

a. cut_1, cut_2 are the cutoff parameters for the ordered logit.

Table 8.2 *Z* Type Distribution

	$A = 1$	$A = 2$	$A = 3$
$\Pr(Z = 1 A)$	0.8347	0.7359	0.6313
	(0.0695)	(0.0265)	(0.0814)
$\Pr(Z = 2 A)$	0.1636	0.2641	0.3687

Based on the estimates in Tables 8.1 and 8.2, I simulate a population of students and report their type distribution in Table 8.3. Of all students, 57% are of ability 1, and only 9% are of ability 3. Conditional on being a type $Z = 2$, the ability distribution first order stochastically dominates that conditional on being a type $Z = 1$. The next three rows show the relationship between family income and ability. The majority of students from both low- and middle-income families are of ability 1; only 32% of students from high-income families are of ability 1. Although ability 3 students are in the minority at all family income levels,

⁶¹In line with the finding from this paper, Dale and Krueger (2002) also find that "a more selective school is not the income-maximizing choice for all students."

their fraction goes up steeply with family income levels. Finally, the last three rows illustrate how noisy *SAT* is as a measure of ability. The ability distribution among *SAT* 1 students is distinctively different from that among higher-*SAT* students: over 88% of *SAT* 1's are of ability 1 and fewer than 1% of them are of ability 3. Between *SAT* 2 and *SAT* 3 students, the ability difference is less obvious. *SAT*, as a noisy measure of student ability, is more powerful in distinguishing ability 1 students from the others, but less so in distinguishing between ability 2 and ability 3 students.

Table 8.3 Ability Distribution: Simulation

%	$A = 1$	$A = 2$	$A = 3$
All	57.2	33.9	8.9
$Z = 1$	60.9	31.9	7.2
$Z = 2$	43.5	41.3	15.2
$Inc = 1$	76.7	20.5	2.8
$Inc = 2$	59.7	33.4	6.9
$Inc = 3$	32.4	48.3	19.3
$SAT = 1$	88.6	10.7	0.7
$SAT = 2$	28.3	58.3	13.4
$SAT = 3$	12.7	57.8	29.5

5.1.3 Application Costs and Financial Aid

Table 9 confirms the hypothesis that applications are costly for students. The cost for the first application is \$6,477, which is higher than the annual tuition of public colleges. As the number of applications increases, the marginal cost rapidly decreases, suggesting the existence of some economies of scale. Given the magnitude of the estimated application cost, it is clear that most of the application cost comes not from application fees, but probably from the time, effort and psychological burdens involved in the application process.⁶²

Table 9 Application Costs

	$n = 1$	$n = 2$	$n = 3$	$n = 4$
$C(n)$	6477.40	7977.17	8335.54	8589.00
	(323.92)	(188.42)	(202.62)	(213.22)

Table 10 displays the estimated parameters for the Tobit specifications of financial aid.⁶³ The left panel reports parameter estimates for general aid. Being black and having higher

⁶²Understanding these costs is an important research question, but is not pursued in this paper.

⁶³The explanatory variables are chosen based on published financial aid policies and on Tobit regressions using only financial aid data: insignificant regressors are omitted. The results reported in Table 10 are estimated jointly with other student-side parameters via SMLE.

SAT scores increases one's expected financial aid, while higher family income and/or assets reduces it. These patterns also hold for college-specific aid, as shown in the right panel. The effect of *SAT* is greater in private colleges than in public colleges. Having siblings in college at the time of application also increases college-specific financial aid. Top colleges are less generous in giving financial aid, especially Group 1, although it charges the highest tuition. By contrast, Group 3 is most generous in giving financial aid. The last row shows that there is significant variation in the final realization of financial aid.

Table 10

	General aid		College-Specific Aid	
	Coefficient	Std. Error	Coefficient	Std. Error
Constant	-4907.82	(817.06)	-13664.32	(1756.28)
Black	1490.72	(915.24)	3277.25	(1033.22)
Family Income	-0.0253	(0.0107)	-0.0461	(0.0092)
Family Assets	-0.0041	(0.0027)	-0.0045	(0.0024)
<i>SAT</i> = 2	3993.10	(854.49)	8141.64	(1837.64)
<i>SAT</i> = 3	6081.56	(1079.32)	15227.48	(1843.60)
Sibling in College ^a			4336.62	(897.90)
(<i>SAT</i> = 2) $\times$ <i>public</i>			-4068.05	(2487.06)
(<i>SAT</i> = 3) $\times$ <i>public</i>			-7821.93	(2563.67)
Group 2			3993.83	(2870.72)
Group 3			9511.52	(1811.92)
Group 4			6854.97	(2278.03)
σ_η	8034.08	(169.34)	9758.76	(285.86)

a. Whether the student has some siblings in college at the time of application.

5.2 College-Side Parameter Estimates

This subsection reports the parameter estimates from the second-step SMDE.⁶⁴ Table 11.1 reports parameter estimates for signal distribution conditional on ability. By sending out signals, the highest ability students can successfully distinguish themselves from the others: they are much more likely to send the highest signal, and almost never send out the lowest signal. Ability 2 students are most likely to send a medium signal, and they distinguish themselves from ability 1 students primarily by their reduced probability of sending out the lowest signal. However, their chance of obtaining the highest signal is almost the same as ability 1 students. As a result, it is hard to distinguish between the two lower-ability types

⁶⁴The overall chi square goodness of fit statistic for SMDE is 41.06 ($\chi^2_{27,0.05} = 40.11$).

by signals. Both *SAT* and signal contain significant amount of noise, reflecting the severity of information friction in the college market.

Table 11.1 Signal Distribution

	$P(s = 1 A)$	$P(s = 2 A)$	$P(s = 3 A)$
$A = 1$	0.2210 (0.0769)	0.3851	0.3939 (0.0954)
$A = 2$	0.0253 (0.0047)	0.5807	0.3940 (0.0810)
$A = 3$	0.000001 (0.0577)	0.2876	0.7124 (0.0575)

The estimated expected capacities are shown in Table 11.2: the more selective colleges and private colleges are smaller than their counterparts. The capacity estimates closely match the capacities observed in the data shown in Table 1.1.

Table 11.2 Capacities

κ_1	κ_2	κ_3	κ_4
0.0096 (0.0015)	0.0459 (0.0013)	0.1082 (0.0009)	0.2456 (0.0021)

Finally, Table 12 shows the results for tuition weights. For both private and public colleges, $m_{j1} > 0$ and $m_{j2} < 0$, which suggests that in addition to using their tuition to compete for better students, colleges have positive but bounded incentives to raise tuition.

Table 12 Tuition Weights

$j \in \{1, 3\}$ private		$j \in \{2, 4\}$ public	
m_{j1}	m_{j2}	m_{j1}	m_{j2}
0.0674 (0.002)	-0.0013 (0.0004)	0.0073 (0.0034)	-0.00063 (0.00015)

Tuition is measured in thousands of dollars.

5.3 Model Fit

Given the parameter estimates, I first fix tuition profile at the data level and simulate the student-side partial equilibrium model (PE) and the application-admission equilibrium model (AE). Then I endogenize tuition and simulate the whole subgame perfect Nash equilibrium model (SPNE).⁶⁵

⁶⁵This section shows model fits for the whole sample. Model fits by race, by *SAT* and by family income are also good, and are available from the author on request.

Table 13.1: Model vs. Data
Distribution of Portfolio Sizes (%)

Size	Data	PE	AE	SPNE
0	54.6	54.9	55.1	55.7
1	30.9	29.6	30.9	31.5
2	11.2	11.8	10.7	9.6
3	2.9	3.3	3.0	2.9
4	0.3	0.2	0.3	0.2
χ^2 Stat		2.95	0.47	5.64
PE: Partial Equilibrium Model				
AE: Application-Admission Equilibrium				
SPNE: Market Equilibrium Model				
$\chi^2_{4,0.05} = 9.49$				

Table 13.1 shows the fit for the distribution of portfolio sizes: all three models fit the data well, with SPNE slightly understating the fraction of multiple applications. Table 14.1 displays the fit of application and admissions rates among applicants. The first set of rows show that all three models closely match application rates, except that the SPNE model under-predicts the application rate for Group 4. The fit for admissions rates is shown in the second set of rows: PE closely matches the admissions rates for all groups. AE and SPNE under-predict the admissions rate for Group 1 and over-predict that for Group 3. Table 15 displays the fits of student allocation. The first set of columns shows the allocation for all students, and the second set of columns shows that for students with at least one admission: all models closely fit the allocation patterns, with SPNE fit being the best.

Finally, Table 16.1 contrasts SPNE predicted tuition levels with the data. The model fits Group 4's tuition almost perfectly, but it under-predicts College 2's tuition and over-predicts College 3's tuition by about 10%.⁶⁶

⁶⁶The deviation of the SPNE tuition from data tuition comes mainly from the SPNE structure. Table 16.2 in the online appendix shows each college's tuition as the best response to others' equilibrium (data) tuition (i.e., the fit for the third-step estimation), which closely matches the data.

Table 14.1 Model vs. Data

Application & Admission: All Applicants (%)

Application Rate	Data	PE	AE	SPNE
Group 1	7.4	7.6	7.1	7.4
Group 2	19.8	21.1	19.9	20.2
Group 3	40.3	41.4	41.2	41.9
Group 4	72.0	72.5	70.8	67.0*
Admission Rate				
Group 1	58.2	54.2	44.1*	43.6*
Group 2	76.4	80.2	81.9	82.0
Group 3	91.7	90.9	95.3*	98.6*
Group 4	95.7	95.0	95.0	97.1

* $\chi^2 > \chi^2_{1,0.05}$

Table 15 Model vs. Data

Final Allocation of Students (%)

	All Students				Students With Some Admission			
	Data	PE	AE	SPNE	Data	PE	AE	SPNE
College 1	1.0	1.1	1.0	1.0	2.2	2.7	2.2	2.2
College 2	4.6	4.5	4.3	4.5	10.6	10.6	10.1	10.5
College 3	11.2	10.7	11.3	11.1	25.6	24.9	26.4	25.8
College 4	24.4	23.5	24.0	24.3	55.7	54.8	55.9	56.3
Outside	58.8	60.2	59.4	59.1	6.0	7.0	5.3	5.1
χ^2 Stat.		2.11	0.54	0.12		1.93	1.54	1.45

 $\chi^2_{4,0.05} = 9.49$

Table 16.1: Model vs. Data

Tuition

	Group 1	Group 2	Group 3	Group 4
Data	27009	5347	17201	3912
SPNE	26162	4555	19173	3925

6 Counterfactual Experiments

With the estimated model, which fits the data reasonably well, I conduct three counterfactual experiments. Comparisons are made between the baseline SPNE and the new SPNE,

simulated using the same set of random draws.⁶⁷

6.1 Perfect Signals

To quantify the impact of incomplete information on the equilibrium, I conduct a counterfactual experiment where signals measure student ability perfectly, i.e., for all A , $P(s = A|A) = 1$.^{68,69}

Table C1.1 Perfect Signals
Distribution of Portfolio Sizes

	<i>Size = 0</i>	<i>Size = 1</i>	<i>Size = 2</i>	<i>Size = 3</i>	<i>Size = 4</i>
Base SPNE	55.7	31.5	9.6	2.9	0.2
New SPNE	57.0	34.3	7.7	0.9	0.1

Table C1.1 contrasts distributions of portfolio sizes. Perfect signaling eliminates the admission uncertainty in most cases and enables students to target their applications more effectively.⁷⁰ Students with no chance of admission are discouraged from applying at all; hence fewer students apply. Moreover, when admission is certain, multiple applications remain meaningful only as a way to guard against ex post shocks; this leads to fewer applications sent by applicants.

Table C1.2 shows changes in admissions rates: as student applications become better targeted, colleges face only well-qualified applicants and all admissions rates increase to near 100%, with Group 1's admissions rate being the highest. Obviously, in this case, "selectivity" as reflected by admissions rates bears no indication about a college's quality, as measured by student ability, as is shown in Table C1.3. The perfect ability measure enables the top groups to fill their capacities with (almost) only the highest-ability students. The lower-ranked groups, although losing some of the highest-ability students, are (almost) free of the lowest-ability students. As a result, average student ability increases for all college groups.

⁶⁷In simulating the baseline model and the counterfactual experiments, I tried a wide range of initial guesses in my search for equilibrium. For each model, I find only one equilibrium.

⁶⁸With perfect signals, SAT no longer affects any decision.

⁶⁹The results from this experiment serve as an upper bound on the impacts of policies that aim to better measure student abilities.

⁷⁰With perfect signals, students would face admission probabilities of either 1 or 0 in most cases. But a student is still subject to rationing if a college's remaining capacity cannot accommodate all applicants in her ability group.

Table C1.2 Perfect Signals

Admission Rates				
%	Group 1	Group 2	Group 3	Group 4
Base SPNE	43.6	82.0	98.6	97.1
New SPNE	99.3	97.4	97.4	98.2

Table C1.3 Perfect Signals

Ability Distribution Within Each Destination					
%	Group 1	Group 2	Group 3	Group 4	Outside
Base SPNE					
$A = 1$	3.7	0.2	7.4	0.9	94.9
$A = 2$	6.5	15.7	80.9	91.9	3.0
$A = 3$	89.9	84.1	11.7	7.2	2.1
New SPNE					
$A = 1$	0.0	0.0	1.8	0.0	96.4
$A = 2$	0.0	0.2	86.6	94.0	2.4
$A = 3$	100.0	99.8	11.6	6.0	1.2

Table C1.4 shows the changes in tuition under the new SPNE. Given perfect signals, colleges no longer need to use tuition as a screening tool. All colleges except Group 1 lower their tuition. Relative to the number of students with highest ability and a strong preference for Group 1, the slots in Group 1 are still scarce. When the signal is perfect, not only does Group 1 admit only the highest-ability students, but it also charges higher tuition. Other colleges do not enjoy the same preferable market position: the perfect signal drives up their competition for better students, which drives down their tuition.

Table C1.4 Perfect Signals

Tuition				
	Group 1	Group 2	Group 3	Group 4
Base SPNE	26162	4555	19173	3925
New SPNE	26409	3307	16579	2956

Finally, changes in student welfare are reported in Table C1.5. The lowest ability students lose significantly, since they are denied admission to almost any college. However, all other students gain, and the highest-ability students benefit the most. On average, student welfare increases by 6%, 24% of which comes directly from tuition changes.

Table C1.5 Perfect Signals

Mean Student Welfare				
	Base SPNE (\$)	New SPNE(\$)	Change (\$)	Change (%)
All	41402	43860	2458	5.94
$A = 1$	677	162	-515	-76.07
$A = 2$	98630	103373	4743	4.81
$A = 3$	84673	97453	12780	15.09

6.2 Funding Cuts

Public colleges rely on government funding as a major source of revenue, and government funding is subject to shocks. For example, in the recent government budget crisis, by September 2009, at least 34 states had implemented funding cuts to public colleges and universities.⁷¹ To balance their budgets, public colleges would be under the pressure to increase their tuition.⁷² The second experiment examines the equilibrium effects of such funding cuts on the college market. I fix all the other parameters at their original levels and increase m_1 for public colleges by 10%, thus pushing public colleges to increase tuition.

Table C2.1 Funding Cuts

Tuition				
	Group 1	Group 2	Group 3	Group 4
Base SPNE	26162	4555	19173	3925
New SPNE	26401	5034	19355	4801

The new equilibrium tuition levels are shown in Table C2.1. Top public colleges (Group2) increase their tuition by about 10%, and lower-ranked public colleges (Group 4) increase their tuition by 22%. The overall increase in public tuition, weighted by enrollment, is 20%. More interestingly, we would like to see how private colleges adjust their tuition. Whether they would increase their tuition is not clear a priori because they care about both tuition revenue and student ability. In this particular case, private colleges also increase tuition, but by less than 1%. Two factors help explain this result: first, higher tuition increases revenue. Second, although lower tuition would help private colleges attract more able students from public colleges, it also attracts less able students, which could lead to a lower-quality student body.

⁷¹For example, the University of California and California State University received 20% less state funding in 2009 than they did in 2007.

⁷²There may be other actions by public colleges in response to funding cuts. This experiment assumes that funding cuts **directly** affect only tuition policy, which is plausibly one of the most important channels. Caveat should be taken as other direct effects are assumed away.

Table C2.2 reports changes in student welfare: all students lose, and the mean welfare decreases by \$700. If the government uses the increased public tuition revenue on a one-for-one basis to save on education expenses, it could save \$234 per student, which is only 1/3 of the welfare loss suffered by students.

Table C2.2 Funding Cuts

	Mean Student Welfare		
	Base SPNE	New SPNE(\$)	Change (\$)
All	41402	40703	-699
$A = 1$	677	649	-28
$A = 2$	98630	97018	-1612
$A = 3$	84673	83149	-1524

6.3 Creating More Opportunities

Finally, I use the model to examine to what extent the government can further expand college access by increasing the supply of colleges. I increase the capacity of the lower-ranked public colleges (Group 4) by growing magnitudes while keeping the capacities of other groups fixed.⁷³ The response of college enrollment to the increase in supply is shown in Figure 3. At the beginning, there is a one-to-one response of college enrollment to the increase in supply. Then, enrollment reaches a satiation point where there is neither excess demand nor excess supply of colleges in Group 4 and the equilibrium outcomes remain the same thereafter. The following tables report the case when Group 4's supply is at the satiation point.

Table C3.1 reports changes in tuition. To attract enough students, Group 4 cuts its tuition from \$3,925 to an almost negligible level of \$136. Its private counterpart, Group 3, also lowers its tuition by about 9%.⁷⁴ However, the two top groups increase their tuition. To better understand the difference in colleges' tuition adjustments, we need to jointly consider their reactions in tuition and admission policies.

Table C3.1 Increasing Supply

	Tuition			
	Group 1	Group 2	Group 3	Group 4
Base SPNE	26162	4555	19173	3925
New SPNE	27549	6473	17394	136

⁷³Similar results hold in analogous experiments with Group 3's capacity. I increase the supply of lower-ranked colleges because they are most relevant to college access.

⁷⁴Colleges do not have to fill their capacities, and they can charge high tuition and leave some slots vacant. However, under the current situation, it is not optimal for them to do so.

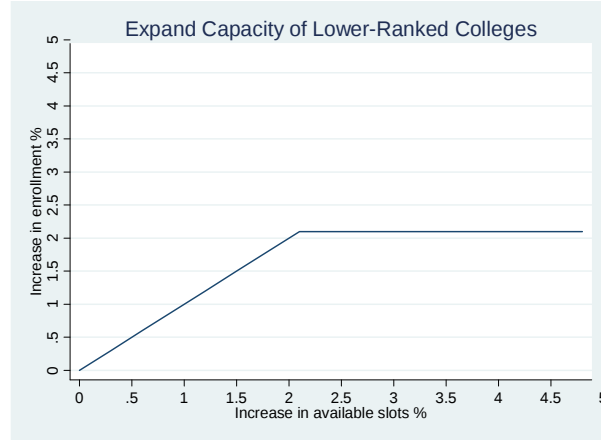


Figure 1: Enrollment & Expansion of Lower-Ranked Groups

Table C3.2 Increasing Supply
Admission Rates

%	Group 1	Group 2	Group 3	Group 4
Base SPNE	43.6	82.0	98.6	97.1
New SPNE	47.7	99.0	99.1	100.0

Table C3.3 Increasing Supply
Attendance

%	Base SPNE	New SPNE	All Open&Free
All	40.9	43.0	51.1
$A = 1$	1.9	3.2	14.9
$A = 2$	94.7	97.7	99.4
$A = 3$	86.4	90.3	98.6

Table C3.2 indicates that admissions rates increase in all colleges and reach (almost) 100% except for Group 1. The major driving forces for the increased admissions rates are likely to differ across college groups. For lower-ranked groups, higher admissions rates and lower tuition reflect their efforts to enroll enough students. Top groups increase their admissions rates mainly because they are faced with a better self-selected applicant pool: the increased tuition in top groups push, and the tuition and admissions policies in lower-ranked colleges pull lower-ability applicants toward lower-ranked groups.

Table C3.3 shows the allocation effect. The first row displays the attendance rate over all students: regardless of the 100% admissions rate and the dramatically lowered tuition in Group 4, only 2.1% more students are drawn into colleges. Since the supply of colleges in

Group 4 exceeds demand if its capacity is further increased, this 2.1% increase represents the upper limit to which the government can increase college attendance by increasing the supply of Group 4 colleges. To further understand these equilibrium results, I conduct a partial equilibrium experiment where all colleges are open and free, and the attendance rate is reported in the last column of Table C3.3: only 51%, or 10% more students, would attend colleges under this condition. Therefore, neither college capacity nor tuition is a major barrier to college access. A vast majority of students who do not attend colleges under the base SPNE prefer the outside option over any college option. Among them, most are of low ability. In fact, as indicated in the last three rows of Table C3.3, only 2% of the lowest-ability students attend college in the base SPNE, and fewer than 15% of them would attend college even if colleges were free and open. In contrast, the majority of students of higher ability attend college in the base SPNE, and almost all of them would attend college if colleges become free and open. The major limit on college access, therefore, is ability and associated preferences.⁷⁵

7 Conclusion

In this paper, I have developed and structurally estimated an equilibrium model of the college market that incorporates tuition setting, applications, admissions and enrollment. In the model, students are heterogeneous in their abilities and preferences. They face uncertainty and application costs when making their application decisions. Colleges, observing only noisy measures of student ability, compete for more able students via tuition and admissions policies. I have estimated the structural model via a three-step estimation procedure to cope with the complications caused by potential multiple equilibria. The empirical results suggest that the model closely replicates most of the patterns in the data.

The estimated structural model has been used to conduct three counterfactual experiments that examine, respectively, the distortion imposed on the market by incomplete information, the equilibrium impacts of funding cuts to public colleges, and the extent to which the government can further expand college access by increasing the supply of lower-ranked colleges. The results suggest that 1) neither tuition cost nor college capacity is a major obstacle to college access, 2) a large fraction of students, mainly low-ability students, prefer the outside option over any college option, and 3) expanding the supply of colleges only draws at most 2.1% more students into colleges.

The methods developed in this paper and the main empirical findings are promising for

⁷⁵Similar conclusions are drawn in earlier studies. See, for example, Cameron and Heckman (1998) and Keane and Wolpin (2001).

future research. Building on Epple, Romano and Sieg (2006) and this paper, a model that endogenizes applications, admissions and financial aid would provide a more comprehensive view of the college market. Building on Arcidiacono (2005) and this paper, a model that studies the strategic interactions between colleges and students and links them to students' labor market outcomes will also be an important extension.

Another extension is to control for additional sources of observed heterogeneity, such as minority status. There are different ways to incorporate such heterogeneity into the model. For example, affirmative action, in terms of more preferable admissions rates for minority groups, may result from colleges' pursuit of racial diversity, or race-specific ability distributions, or some combination of both.⁷⁶ All of these conjectures would lead to different equilibrium results, which could be tested against the data. The ability to distinguish between these different sources would allow for a better evaluation of affirmative action.

Finally, endogenizing capacity constraints and studying the long-run equilibrium would lead to a better understanding of the trend of college tuition and attendance. One approach to implement this extension is to introduce a cost function for college education, assuming free entry to the market. Equilibrium of the model would then depend on the form of the cost function. Estimation of such a model would require additional data on college expenses and non-tuition revenues, as well as application and admissions data over multiple years.

References

- Arcidiacono, Peter. 2005. "Affirmative Action in Higher Education: How Do Admission and Financial Aid Rules Affect Future Earnings?" *Econometrica*, 73(5): 1477-1524.
- Arcidiacono, Peter and Sean Nicholson. 2005. "Peer effects in medical school." *Journal of Public Economics*, 89(2-3): 327-350
- Avery, Christopher, Andrew Fairbanks and Richard Zeckhauser. 2003. *The Early Admissions Game: Joining the Elite*. Cambridge, MA: Harvard University Press.
- Avery, Christopher, Jonathan Levin. 2010. "Early Admissions at Selective Colleges." *American Economic Review*, 100(6): 1-34.
- Bowen, William and Derek Bok. 1998. *The Shape of the River: Long-Term Consequences of Considering Race in College and University Admissions*. Princeton, NJ: Princeton University Press.

⁷⁶For example, on the one hand, colleges may have race-specific capacity constraints that favor black students over white students of the same expected ability. On the other hand, given the same *SAT* and signal, black students may be more able than white students.

- Chade, Hector, Gregory Lewis and Lones Smith. 2009. "A Supply and Demand Model of the College Admissions Problem." Social Science Research Network Working Paper 135834.
- Cameron, Stephen and James Heckman. 1998. "Life Cycle Schooling and Dynamic Selection Bias: Models and Evidence for Five Cohorts of American Males." *Journal of Political Economy*, 106(2): 262-333.
- Dale, Stacy and Alan Krueger. 2002. "Estimating The Payoff To Attending A More Selective College: An Application Of Selection On Observables And Unobservables." *The Quarterly Journal of Economics*, 117(4): 1491-1527.
- Epple, Dennis, Richard Romano and Holger Sieg. 2006. "Admission, Tuition, And Financial Aid Policies in The Market For Higher Education." *Econometrica*, 74(4): 885-928.
- Howell, Jessica. 2010. "Assessing the Impact of Eliminating Affirmative Action in Higher Education." *Journal of Labor Economics*, 28(1): 113-166.
- Light, Audrey and Wayne Strayer. 2002. "From Bakke to Hopwood: Does Race Affect College Attendance and Completion?" *The Review of Economics and Statistics*, 85: 34-44.
- McFadden, Daniel. 1989. "A Method of Simulated Moments for Estimation of Discrete Response Models without Numerical Integration." *Econometrica*, 57(5): 995-1026.
- Kane, Thomas. 1998. "Racial and Ethnic Preferences in College Admissions." in *The Black-White Test Score Gap*, ed. Christopher Jencks and Meredith Phillips, 971-996. Washington D.C.: Brookings Institution Press.
- Mailath, George, Masahiro Okuno-Fujiwara and Andrew Postlewaite. 1993. "On Belief Based Refinements in Signaling Games." *Journal of Economic Theory*, 60: 241-276.
- Manski, Charles and David Wise. 1983. *College Choice in America*. Cambridge, MA: Harvard University Press
- Meijer, Erik and Jelmer Ypma. 2008. "A Simple Identification Proof for a Mixture of Two Univariate Normal Distributions." *Journal of Classification*, 25: 113-123.
- Moro, Andrea. 2003. "The Effect of Statistical Discrimination on Black-White Wage Differentials: Estimating a Model with Multiple Equilibria." *International Economic Review*, 44(2): 467-500.
- Nagypál, Eva. 2004. "Optimal Application Behavior with Incomplete Information." mimeo, Northwestern University.

Keane, Michael and Kenneth Wolpin. 2001. "The Effect of Parental Transfers and Borrowing Constraints on Educational Attainment." *International Economic Review*, 42(4): 1051-1103.

Sacerdote, Bruce. 2001. "Peer Effects with Random Assignment: Results from Dartmouth Roommates." *Quarterly Journal of Economics*, 116: 681-704.

Zimmerman, David. 2003. "Peer Effects in Higher Education: Evidence from a Natural Experiment." *The Review of Economics and Statistics*, 85: 9-23.

APPENDIX

A. Model Details:

A1. College Admission Problem: $\alpha_j(s, SAT|t, e_{-j}, Y, d)$ and $\gamma_j(s, SAT|t, e_{-j}, Y, d)$

All objects defined in A1 depend on $\{t, e_{-j}, Y, d\}$. To save notation, the dependence is suppressed. Let $\Pr(\text{accept}|X, SAT, \eta, \zeta, j)$ be the probability that a Group j applicant with characteristics (X, SAT, η, ζ) accepts j 's admission. Let $F(X, \eta, \zeta|s, SAT, j)$ be the distribution of (X, η, ζ) conditional on (s, SAT) and application to j . The probability that an applicant with (s, SAT) accepts j 's admission is:

$$\alpha_j(s, SAT) = \int \Pr(\text{accept}|X, SAT, \eta, \zeta, j) dF(X, \eta, \zeta|s, SAT, j).$$

Let $\Pr(O_{-j}|A, SAT) \equiv \prod_{l \in O \setminus j} p_l(A, SAT) \prod_{k \in Y \setminus O} (1 - p_k(A, SAT))$ be the probability of admission set O for a student with (A, SAT) , with college j admitting her for sure,

$$\Pr(\text{accept}|X, SAT, \eta, \zeta, j) = \sum_{O_{-j} \subseteq Y(X, SAT) \setminus \{j\}} \Pr(O_{-j}|A, SAT) I(j = d(X, SAT, \eta, \zeta, O)).$$

That is, the student will accept j 's admission if j is the best post-application choice for her. The distribution $F(X, \eta, \zeta|s, SAT, j)$ is given by

$$dF(X, \eta, \zeta|s, SAT, j) = \frac{P(s|A) I(j \in Y(X, SAT)) dF(X, \eta, \zeta|SAT)}{\int P(s|A) I(j \in Y(X, SAT)) dF(X, \eta, \zeta|SAT)},$$

where $F(X, \eta, \zeta|SAT) = P(T|SAT, B)G(\epsilon, \eta, \zeta)H(B|SAT)$ is exogenous. Finally, the expected ability of applicant (s, SAT) conditional on acceptance is

$$\gamma_j(s, SAT) = \frac{\int A \times \Pr(\text{accept}|X, SAT, \eta, \zeta, j) dF(X, \eta, \zeta|s, SAT, j)}{\alpha_j(s, SAT)}.$$

A2. Proof of existence in a simplified model.

Assume there are two colleges $j \in \{1, 2\}$, a continuum of students divided into two ability levels. The utility of the outside option is normalized to 0. The utility of attending college 1 is $u_1(A)$ for all with ability A , and that of attending college 2 is $u_2(A) + \epsilon$, where ϵ is i.i.d. idiosyncratic taste. There are two SAT levels and two signal levels. There is no ex-post shock. Some notations to be used: for an (A, SAT) group, let the fraction of students that do not apply to any college be $\theta_{A,SAT}^0$, the fraction of those applying to college j only be $\theta_{A,SAT}^j$ and the fraction applying to both be $\theta_{A,SAT}^{12}$. For each (A, SAT) group, $\theta_{A,SAT} \in \Delta$, a 3-simplex. For all four (A, SAT) groups, $\theta \in \Theta \equiv \Delta^4$. On the college side, each college chooses admissions policy $e_j \in [0, 1]^4$, where 4 is the number of (s, SAT) groups faced by the college.

Proposition 1 *For any given tuition profile t , an application-admission equilibrium exists.*

Proof. Step 1: The application-admission model can be decomposed into the following sub-mappings:

Taking the distribution of applicants, and the admissions policy of the other college as given, college j 's problem (6) can be viewed as the sub-mapping

$$M_j : \Theta \times [0, 1]^4 \rightrightarrows [0, 1]^4,$$

for $j = 1, 2$. Taking college admissions policies as given, the distribution of students is obtained via the sub-mapping

$$M_3 : [0, 1]^4 \times [0, 1]^4 \rightarrow \Theta.$$

An equilibrium is a fixed point of the mapping:

$$\begin{aligned} M & : \quad \Theta \times [0, 1]^4 \times [0, 1]^4 \rightrightarrows \Theta \times [0, 1]^4 \times [0, 1]^4 \\ \text{s.t. } \theta & \in M_3(e_1, e_2) \\ e_j & \in M_j(\theta, e_k) \quad j, k \in \{1, 2\}, j \neq k. \end{aligned}$$

Step 2: Show that Kakutani's Fixed Point Theorem applies in mapping M and hence an equilibrium exists.

- 1) The domain of the mapping, being the product of simplexes, is compact and non empty.
- 2) It can be shown that the correspondence $M_j(\cdot, \cdot)$ is compact-valued, convex-valued and upper-hemi-continuous, for $j = 1, 2$. In particular, the (s, SAT) 'th component of $M_j(\theta, e_k)$ is characterized by (7) and (8), where $\gamma_j(s, SAT) + m_{j1}t_j + m_{j2}t_j^2 - \lambda_j$ is continuous in (θ, e_k) .
- 3) Aggregate individual optimization into distribution of students θ .

Generically, each student has a unique optimal application portfolio as the solution to (5). For given (A, SAT) , there exist $\epsilon^*(e) \geq \epsilon^{**}(e)$, both continuous in e , such that:

For $\epsilon \geq \epsilon^*(e)$, $Y(A, SAT, \epsilon) = \begin{cases} \{2\} & \text{if } C(2) - C(1) > k_1(e) \\ \{1, 2\} & \text{otherwise} \end{cases}$;

for $\epsilon \in [\epsilon^{**}(e), \epsilon^*(e))$, $Y(A, SAT, \epsilon) = \{1, 2\}$; and

for $\epsilon < \epsilon^{**}(e)$, $Y(A, SAT, \epsilon) = \begin{cases} \{1\} & \text{if } C(1) \leq k_2(e) \\ \emptyset & \text{otherwise} \end{cases}$,

where $k_1(e)$ and $k_2(e)$ are continuous in e . Therefore, the (A, SAT) population can be mapped into a distribution $\theta_{A, SAT} \in \Delta$, and this mapping is continuous in e . Because the mapping from $[0, 1]^4 \times [0, 1]^4$ into the individual optimal portfolio is a continuous function, and the mapping from the individual optimization to Θ is continuous, the composite of these two mappings, M_3 , is single-valued and continuous.⁷⁷

Given 1)-3), Kakutani's Fixed Point Theorem applies.

⁷⁸ ■

Since for every t , $AE(t)$ exists in the subsequent game, an SPNE exists if a Nash equilibrium exists in the tuition setting game. Let $\bar{t}_j$ denote some large positive number, such that for any t_{-j} , the optimal $t_j < \bar{t}_j$. $\bar{t}_j$ exists because the expected enrollment, hence college j 's payoff, goes to 0 as t_j goes to ∞ . Define the strategy space for college j as $[0, \bar{t}_j]$, which is nonempty, compact and convex. The objective function of college j is continuous in t , since the distribution of applicants, and hence the total expected ability, is continuous in t . Given certain regularity conditions, the objective function is also quasi-concave in t_j . The general existence proof for Nash equilibrium applies.

B. Data Details

Empirical Definition of Early Admission:

1) Applications were sent earlier than Nov. 30th, for attendance in the next fall semester and

2) The intended college has early admissions/ early decision/ rolling admissions/ priority admissions policy,⁷⁹ and

3) Either *a.* one application was sent early and led to an admission or

b. some application(s) was (were) sent early but rejected, and other application(s) was (were) sent later.

⁷⁷In the case of four schools, ϵ becomes a 3-dimension vector, as are the cutoff tastes. To show continuity, we change one dimension of ϵ at a time while keeping the other dimensions fixed.

⁷⁸When there are $J > 2$ schools, Step 1 of the proof can be easily extended. In Step 2, ϵ , and hence the cutoffs, will be of $j - 1$ dimensions. Obtaining an analytical solution to these cutoffs becomes much more challenging.

⁷⁹The data source for college early admission programs is 1) Christopher et. al. (2003), and 2) web information posted by individual colleges.

C. Detailed Functional Forms

C1. Conditional Ability Distribution: for $a = 1, 2, 3$

$$\Pr(A_i = a) = \frac{1}{1 + e^{-cut_a + \alpha_1 y_i + \alpha_2 I(SAT_i=2) + \alpha_3 I(SAT_i=3)}} - \frac{1}{1 + e^{-cut_{a-1} + \alpha_1 y_i + \alpha_2 I(SAT_i=2) + \alpha_3 I(SAT_i=3)}}$$

where y_i denotes family income of i , $cut_0 = -\infty$ and $cut_3 = +\infty$.

C2. Financial Aid Functions:

1) General Aid:

$$\begin{aligned} f_0(SAT_i, B_i) &= \beta_0^0 + \beta_1^0 I(race_i = black) + \beta_2^0 I(SAT_i = 2) + \beta_3^0 I(SAT_i = 3) \\ &\quad + \beta_4^0 y_i + \beta_5^0 asset_i \\ f_{0i} &= \max\{f_0(SAT_i, B_i) + \eta_{0i}, 0\}, \end{aligned}$$

where $\eta_{0i} \sim i.i.d.N(0, \sigma_{f_0}^2)$.

2) College-Specific Financial Aid:

$$f_j(SAT_i, B_i) =$$

$$\begin{aligned} &\beta_0^1 + \beta_1^1 I(race_i = black) + \beta_2^1 I(SAT_i = 2) + \beta_3^1 I(SAT_i = 3) + \beta_4^1 y_i + \beta_5^1 asset_i \\ &\quad + \beta_6^1 I(nsib > 0) + \beta_7^1 I(SAT_i = 2) I(j \in public) + \beta_8^1 I(SAT_i = 3) I(j \in public) \\ &\quad + \beta_9^1 I(j = 2) + \beta_{10}^1 I(j = 3) + \beta_{11}^1 I(j = 4) \end{aligned}$$

$$f_{ji} = \max\{f_j(SAT_i, B_i) + \eta_{ji}, 0\}$$

where $nsib$ denotes the number of siblings in college at the time of i 's application and $\eta_{ji} \sim i.i.d.N(0, \sigma_{f_1}^2)$.

C3. Preferences: $\bar{u}_j(A, Z = 1)$ are fully non-parametric and $\bar{u}_j(A, Z = 2) = \bar{u}_j(A, Z = 1) + \bar{\psi}_j(A)$, with the restriction that $\bar{\psi}_j(1) = \bar{\psi}_j(2)$.⁸⁰

D. Details on Estimation

D1. Details on SMLE:

⁸⁰This restriction is imposed to save the number of parameters. The restricted model cannot be rejected at 10% significance level.

(1) Approximate the following integration via a kernel smoothed frequency simulator ⁸¹

$$\int I(Y_i|T, SAT_i, B_i, \epsilon) I(d_i|O_i, T, SAT_i, B_i, \epsilon, \zeta, \eta) dG(\epsilon, \zeta, \eta). \quad (14)$$

For each student (SAT_i, B_i) , I draw shocks $\{(\epsilon_{ir}, \zeta_{ir}, \eta_{ir})\}_{r=1}^R$ from their joint distribution $G(\cdot)$. These shocks are the same across T for the same student i , but are i.i.d. across students. All shocks are fixed throughout the estimation. Let u_{jir} be the ex-post value of college j for student ir with $(T, SAT_i, B_i, \epsilon_{ir}, \zeta_{ir}, \eta_{ir})$, let $v_{ir} = \max\{0, \{u_{jir}\}_{j \in O_i}\}$, let $V_{ir}(Y)$ be the ex-ante value of portfolio Y for this student, and $V_{ir}^* = \max_{Y \subseteq J} \{V_{ir}(Y)\}$. (14) is then approximated by:

$$\frac{1}{R} \sum_{r=1}^R \frac{\exp[(V_{ir}(Y_i) - V_{ir}^*)/\tau_1]}{\sum_{Y \subseteq J} \exp[(V_{ir}(Y) - V_{ir}^*)/\tau_1]} \frac{\exp[(u_{d_{ir}} - v_{ir})/\tau_2]}{\sum_{j \in O_i} \exp[(u_{jir} - v_{ir})/\tau_2]},$$

where τ_1, τ_2 are smoothing parameters. When $\tau \rightarrow 0$, the approximation converges to the frequency simulator.

(2) Solving the optimal application problem for student $(T, SAT_i, B_i, \epsilon_{ir})$:

$$V_i(Y) = \sum_{O \subseteq Y} \Pr_i(O) E_{(\eta, \zeta)} \max\{u_{0ir}, \{u_{jir}\}_{j \in O}\} - C(|Y|).$$

The Emax function has no closed-form expression and is approximated via simulation. For each $(T, SAT_i, B_i, \epsilon_{ir})$, draw M sets of shocks $\{(\eta_m, \zeta_m)\}_{m=1}^M$. For each of the M sets of $(T, SAT_i, B_i, \epsilon_{ir}, \eta_m, \zeta_m)$, calculate $\max\{u_{0irm}, \{u_{jirm}\}_{j \in O}\}$, where u_{jirm} denotes u_{jir} evaluated at the shock (η_m, ζ_m) . The Emax is the average of these M maximum values.

D2. Details on the Second-Step SMDE:

(1) Targets to be matched: for each of the Groups 2, 3 and 4, there are 9 admissions probabilities to be matched $\{p_j(A, SAT)\}_{(A, SAT) \in \{1,2,3\} \times \{1,2,3\}}$. For Group 1, there are 6 admissions probabilities to be matched. Since no one in $SAT = 1$ group applied to Group 1, $\{p_1(A, SAT = 1)\}_{A \in \{1,2,3\}}$ are fixed at 0. The other four targets are the equilibrium enrollments simulated from the first step. In all, there are 37 targets to be matched using college-side parameters: $\{P(s|A)\}, \{\kappa_j\}_j$, ten of which are free.

(2) Optimal Weighting Matrix:

Let Θ^* be the true parameter values. The first-step estimates $\hat{\Theta}_1$, being MLE, are asymptotically distributed as $N(0, \Omega_1)$. It can be shown that the optimal weighting matrix

⁸¹I describe the situation where I do not observe any information about the student's financial aid. For students with some financial aid information, the observed financial aid replaces the random draw of the corresponding financial aid shock.

for the second-step objective function (13) is $W = Q_1 \Omega_1 Q_1'$, where Q_1 is the derivative of $q(\cdot)$ with respect to $\hat{\Theta}_1$, evaluated at $(\hat{\Theta}_1, \Theta_2^*)$. The estimation of W involves the following steps:

1) Estimate the variance-covariance matrix $\hat{\Omega}_1$: in the case of MLE, this is minus the outer product of the score functions evaluated at $\hat{\Theta}_1$. The score functions are obtained via numerically taking partial derivatives of the likelihood function with respect to each of the first step parameters evaluated at $\hat{\Theta}_1$.

2) Obtain preliminary estimates $\tilde{\Theta}_2 \equiv \arg \min_{\Theta_2} \{q(\hat{\Theta}_1, \Theta_2)' \tilde{W} q(\hat{\Theta}_1, \Theta_2)\}$, where $\tilde{W}$ is any positive-definite matrix. The resulting $\tilde{\Theta}_2$ is a consistent estimator of Θ_2^* .

3) Estimate Q_1 by numerically taking derivative of $q(\cdot)$ with respect to $\hat{\Theta}_1$, evaluated at $(\hat{\Theta}_1, \tilde{\Theta}_2)$. In particular, let Δ_m denote a vector with zeros everywhere but the m 'th entry, which equals a small number ε_m . At each $(\hat{\Theta}_1 + \Delta_m, \tilde{\Theta}_2)$, I simulate the student decision model and calculate the targets for the second-step estimation. Then holding student applications fixed, I solve for college optimal admissions and calculate the distance vector $q(\hat{\Theta}_1 + \Delta_m, \tilde{\Theta}_2)$. The m 'th component of Q_1 is approximated by $[q(\hat{\Theta}_1 + \Delta_m, \tilde{\Theta}_2) - q(\hat{\Theta}_1, \tilde{\Theta}_2)]/\varepsilon_m$.

E. Identification of A Mixture of Two Probits

Assume there are two unobserved types of individuals $A \in \{1, 2\}$, and $\Pr(A = 1) = \lambda$. Let the continuous variable $z \in Z \subseteq R$ be an observed individual characteristics and $f(\cdot)$ be a differentiable function of z . Let $y \in \{0, 1\}$ be the observed discrete choice, which relates to the latent variable y^* in the following way:

$$y(z) = 1 \text{ if only if } y^*(z) \equiv f(z) + u_1 I(A = 1) + u_2 I(A = 2) + \epsilon > 0$$

where $\epsilon \sim i.i.d. N(0, 1)$. The model implies that

$$P(z) \equiv \Pr(y(z) = 1) = \lambda \Phi(f(z) + u_1) + (1 - \lambda) \Phi(f(z) + u_2) \quad (15)$$

Theorem 1 *Assume that 1) $\lambda \in (0, 1)$, 2) there exists an open set $Z^* \subseteq Z$ such that for $z \in Z^*$, $f'(z) \neq 0$. Then the parameters $\theta = (\lambda, u_1, u_2)'$ in (15) are locally identified.*

Proof. The proof draws on the well-known equivalence of local identification with positive definiteness of the information matrix. In the following, I will show the positive definiteness of the information matrix for model (15).

Step 1. Claim: *The information matrix $I(\theta)$ is positive definite if and only if there exist no $w \neq 0$, such that $w' \frac{\partial P(z)}{\partial \theta} = 0$ for all z .*

The log likelihood of an observation (y, z) is

$$L(\theta) = y \ln(P(z)) + (1 - y) \ln(1 - P(z)).$$

The score function is given by

$$\frac{\partial L}{\partial \theta} = \frac{y - P(z)}{P(z)(1 - P(z))} \frac{\partial P(z)}{\partial \theta}.$$

Hence, the information matrix is

$$I(\theta|z) = E \left[\frac{\partial L}{\partial \theta} \frac{\partial L}{\partial \theta'} | z \right] = \frac{1}{P(z)(1 - P(z))} \frac{\partial P(z)}{\partial \theta} \frac{\partial P(z)}{\partial \theta'}.$$

Given $P(z) \in (0, 1)$, it is easy to show that the claim holds.

Step 2. Show $w' \frac{\partial P(z)}{\partial \theta} = 0$ for all $z \implies w = 0$.

$\frac{\partial P(z)}{\partial \theta}$ is given by:

$$\begin{aligned} \frac{\partial P(z)}{\partial \lambda} &= \Phi(f(z) + u_1) - \Phi(f(z) + u_2) \\ \frac{\partial P(z)}{\partial u_1} &= \lambda \phi(f(z) + u_1) \\ \frac{\partial P(z)}{\partial u_2} &= (1 - \lambda) \phi(f(z) + u_2) \end{aligned}$$

Suppose for some w , $w' \frac{\partial P(z)}{\partial \theta} = 0$ for all z :

$$w_1 [\Phi(f(z) + u_1) - \Phi(f(z) + u_2)] + w_2 \lambda \phi(f(z) + u_1) + w_3 (1 - \lambda) \phi(f(z) + u_2) = 0$$

Take derivative with respect to z evaluated at some $z \in Z^*$

$$\begin{aligned} w_1 [\phi(f(z) + u_1) - \phi(f(z) + u_2)] f'(z) + w_2 \lambda \phi'(f(z) + u_1) f'(z) \\ + w_3 (1 - \lambda) \phi'(f(z) + u_2) f'(z) = 0. \end{aligned} \tag{16}$$

Define $\gamma(z) = \frac{\phi(f(z) + u_1)}{\phi(f(z) + u_2)}$, divide (16) by $\phi(f(z) + u_2)$:

$$\begin{aligned} w_1 [\gamma(z) - 1] - w_2 \lambda (f(z) + u_1) \gamma(z) - w_3 (1 - \lambda) (f(z) + u_2) &= 0 \\ \gamma(z) [w_1 - w_2 \lambda (f(z) + u_1)] - [w_1 + w_3 (1 - \lambda) (f(z) + u_2)] &= 0 \end{aligned} \tag{17}$$

Since $\gamma(z)$ is a nontrivial exponential function of z , (17) hold for all $z \in Z^*$ only if both

terms in brackets are zero for each $z \in Z^*$, i.e.

$$\begin{aligned} w_1 - w_2\lambda(f(z) + u_1) &= 0 \\ w_1 + w_3(1 - \lambda)(f(z) + u_2) &= 0. \end{aligned} \tag{18}$$

Take derivative of (18) again with respect to z , evaluated at $z \in Z^*$:

$$\begin{aligned} w_2\lambda f'(z) &= 0 \\ w_3(1 - \lambda)f'(z) &= 0. \end{aligned}$$

Since $\lambda \in (0, 1)$ and $f'(z) \neq 0$ for some z , $w = 0$. ■

F. Additional Tables

F1. College Value: A Simulated Example
Table 7.2 College Value: A Simulated Example

	All	Applicants	Attendees
$\tilde{u}_1(A = 1, Z = 1)$	-234068.4 (115117.4)	85945.8 (30811.5)	99942.4 (34730.2)
$\tilde{u}_1(A = 2, Z = 1)$	-222834.1 (115493.2)	117083.6 (43664.9)	157089.1 (44945.0)
$\tilde{u}_1(A = 3, Z = 1)$	-57699.4 (115636.8)	134435.5 (58596.4)	159033.6 (58609.5)
$\tilde{u}_1(A = 1, Z = 2)$	-74090.3 (115117.4)	108956.4 (50605.7)	126051.4 (50773.6)
$\tilde{u}_1(A = 2, Z = 2)$	-62856.0 (115493.2)	133911.5 (57014.3)	158914.7 (58820.5)
$\tilde{u}_1(A = 3, Z = 2)$	123994.5 (115637.1)	187099.1 (82266.7)	211289.7 (79100.2)

F2. Model Fit

Table 16.2 Tuition Fit in Step-3

	Top Priv.	Top Pub.	Low Priv.	Low Pub.
Data	27009	5347	17201	3912
Best Response	27579	4954	18010	3921

F3. Robustness Check: Counterfactual Experiments With Alternative ω ⁸²

F3.1 Perfect Signal

⁸²This subsection shows the results for $\omega = [1, 1.4, 2]'$. For other ω 's around $[1, 2, 3]$, the results are similarly robust.

Table F3.1.1 Perfect Signals

Distribution of Portfolio Sizes

	<i>Size</i> = 0	<i>Size</i> = 1	<i>Size</i> = 2	<i>Size</i> = 3	<i>Size</i> = 4
Base SPNE	55.9	31.8	9.2	2.9	0.2
New SPNE	57.1	34.5	7.4	1.0	0.1

Table F3.1.2 Perfect Signals

Admission Rates

%	Group 1	Group 2	Group 3	Group 4
Base SPNE	44.1	82.7	99.0	98.2
New SPNE	93.8	97.7	97.5	99.2

Table F3.1.3 Perfect Signals

Ability Distribution Within Each Destination

%	Group 1	Group 2	Group 3	Group 4	Outside
Base SPNE					
$A = 1$	3.4	0.2	7.4	0.8	95.0
$A = 2$	6.2	15.3	81.1	92.1	2.9
$A = 3$	90.4	84.5	11.5	7.1	2.1
New SPNE					
$A = 1$	0.0	0.0	1.8	0.0	96.4
$A = 2$	0.0	0.1	86.6	94.2	2.3
$A = 3$	100.0	99.9	11.5	5.8	1.3

Table F3.1.4 Perfect Signals

Tuition

	Group 1	Group 2	Group 3	Group 4
Base SPNE	26940	4773	19907	4392
New SPNE	27004	3825	17251	3718

Table F3.1.5 Perfect Signals

Mean Student Welfare

	Base SPNE (\$)	New SPNE(\$)	Change (\$)	Change (%)
All	41396	43575	2179	5.3
$A = 1$	670	161	-509	-76.0
$A = 2$	98248	102550	4302	4.4
$A = 3$	84550	95740	11190	13.2

F3.2 Funding Cuts

Table F3.2.1 Funding Cuts

Tuition				
	Group 1	Group 2	Group 3	Group 4
Base SPNE	26940	4773	19907	4392
New SPNE	27103	5394	20095	4821

Table F3.2.2 Funding Cuts

Mean Student Welfare			
	Base SPNE	New SPNE(\$)	Change (\$)
All	41396	40639	-757
$A = 1$	670	647	-23
$A = 2$	98248	96497	-1751
$A = 3$	84550	82903	-1647

F3.3 Creating Opportunity

Table F3.3.1 Increasing Supply

Tuition				
	Group 1	Group 2	Group 3	Group 4
Base SPNE	26940	4773	19907	4392
New SPNE	27534	6890	18176	98

Table F3.3.2 Increasing Supply

Admission Rates				
%	Group 1	Group 2	Group 3	Group 4
Base SPNE	44.1	82.7	99.0	98.2
New SPNE	47.3	95.3	99.8	100.0

Table F3.3.3 Increasing Supply

Attendance Rate	
Base SPNE	New SPNE
40.9	43.0